Thermodynamic Properties of Strongly Interacting Matter

A reminder/summary:

QUARK MATTER 2004

FK, J. Phys. G30 (2004) S887 [hep-lat/0403016]

News from Lattice QCD

On

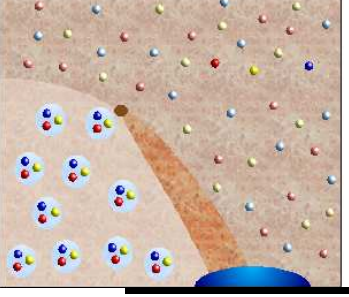
Heavy Quark Potentials and

Spectral Functions of Heavy Quark States

Quark Matter 2004 ...

...topics that could not be discussed:

1) Update on the QCD phase diagram
   - $T_c(\mu)$ and the chiral critical point: quark mass dependence, lattice artefacts
2) QCD equation of state for $\mu > 0$
   - baryon number fluctuations;
   - comparison with resonance gas



Topics from Lattice QCD ...

...to be discussed at SQM2004:

1) Update on the QCD phase diagram
   - $T_c(\mu)$ and the chiral critical point: quark mass dependence, lattice artefacts
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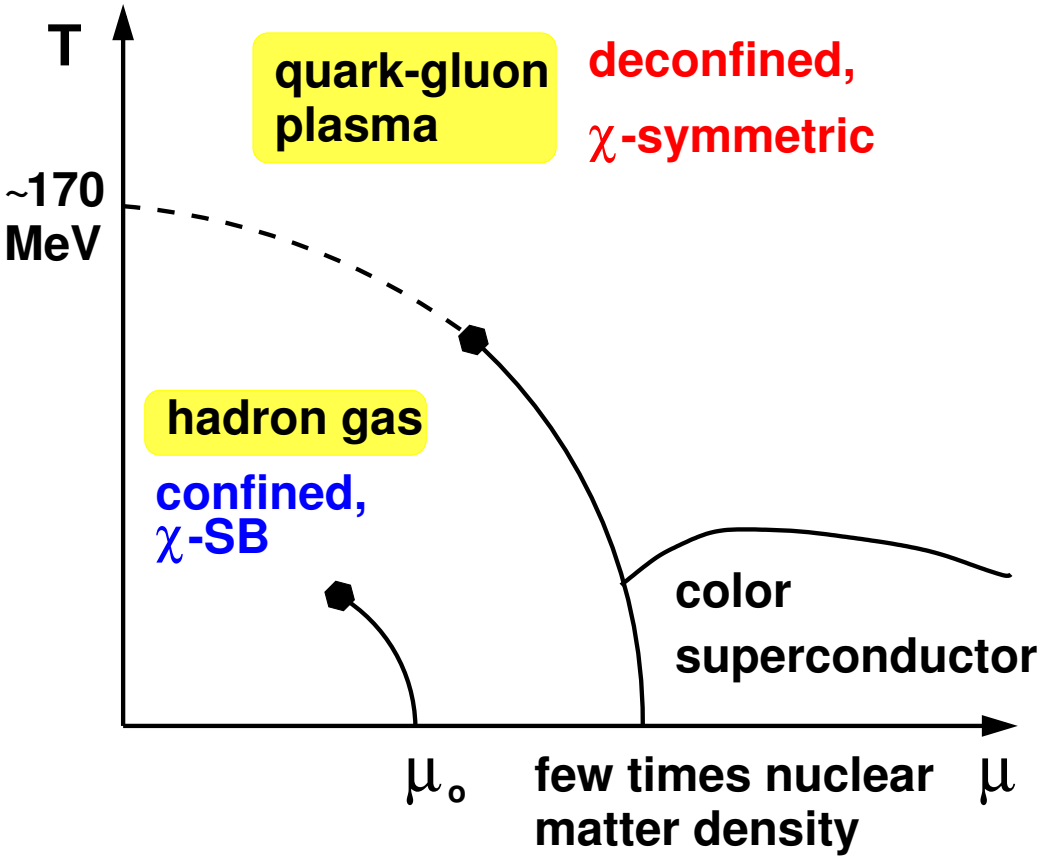
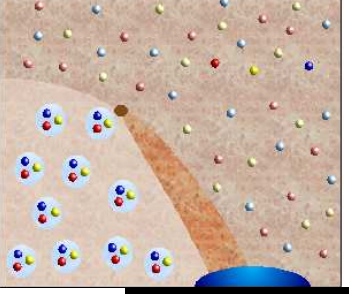
Critical behavior in hot and dense matter: QCD phase diagram

crossover vs.

phase transition





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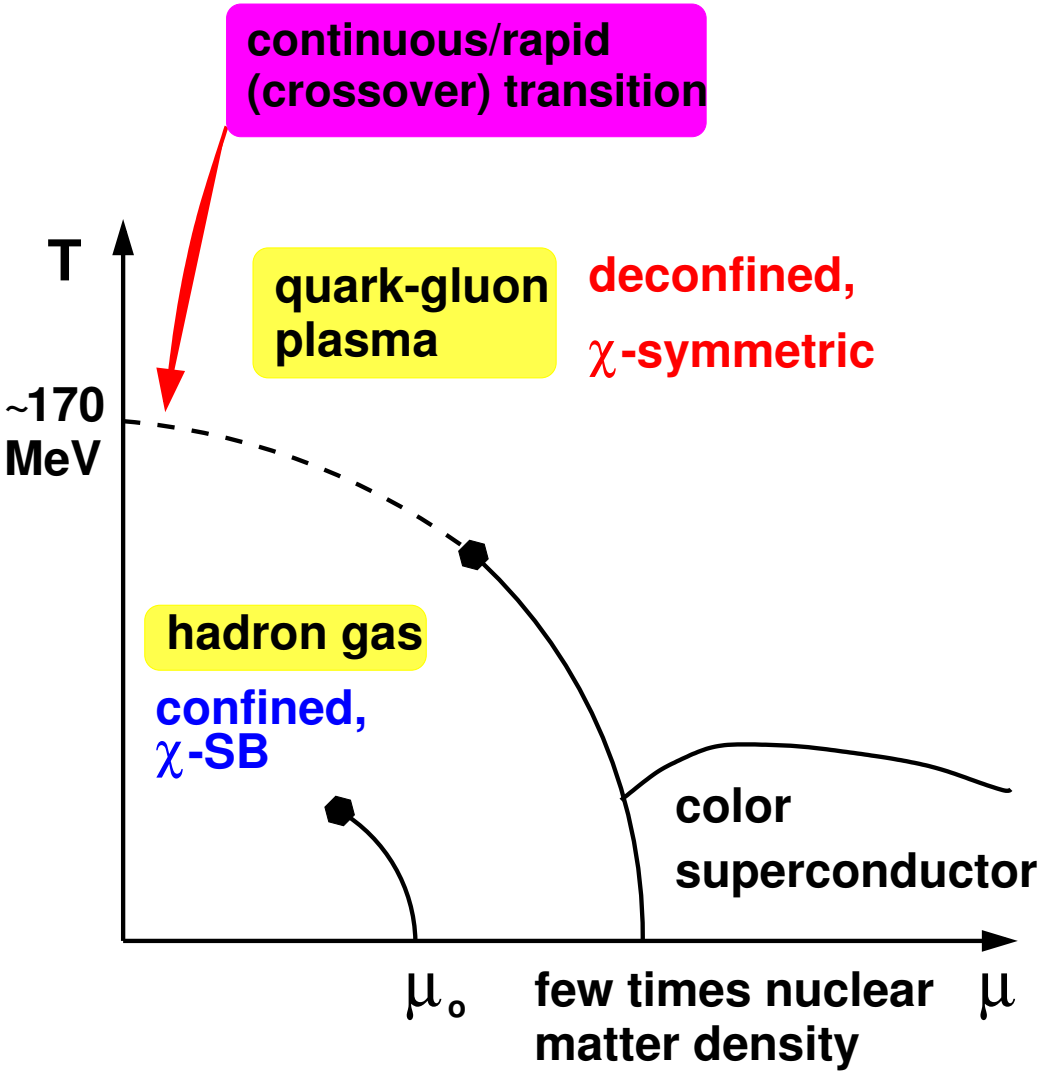
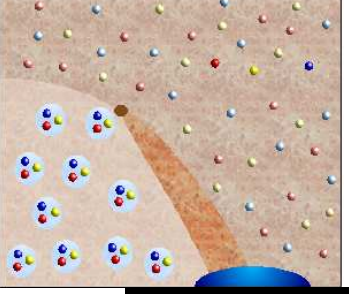


continuous transition for small chemical potential and small quark masses at

$$T_c \simeq 170\, MeV$$
$$\epsilon_c \simeq 0.7\, GeV/fm^3$$



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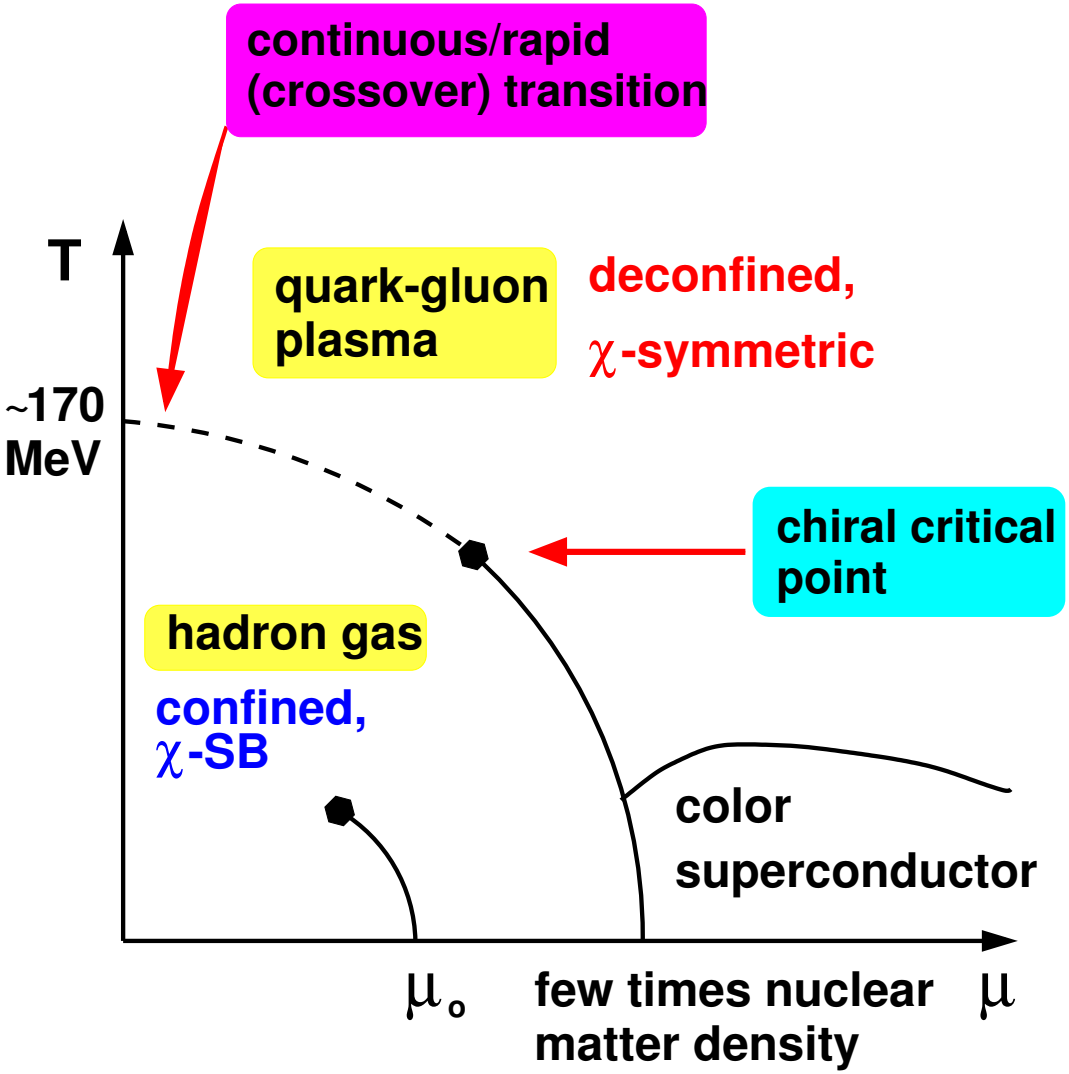

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2nd order phase transition; Ising universality class

$T_c(\mu)$ under investigation

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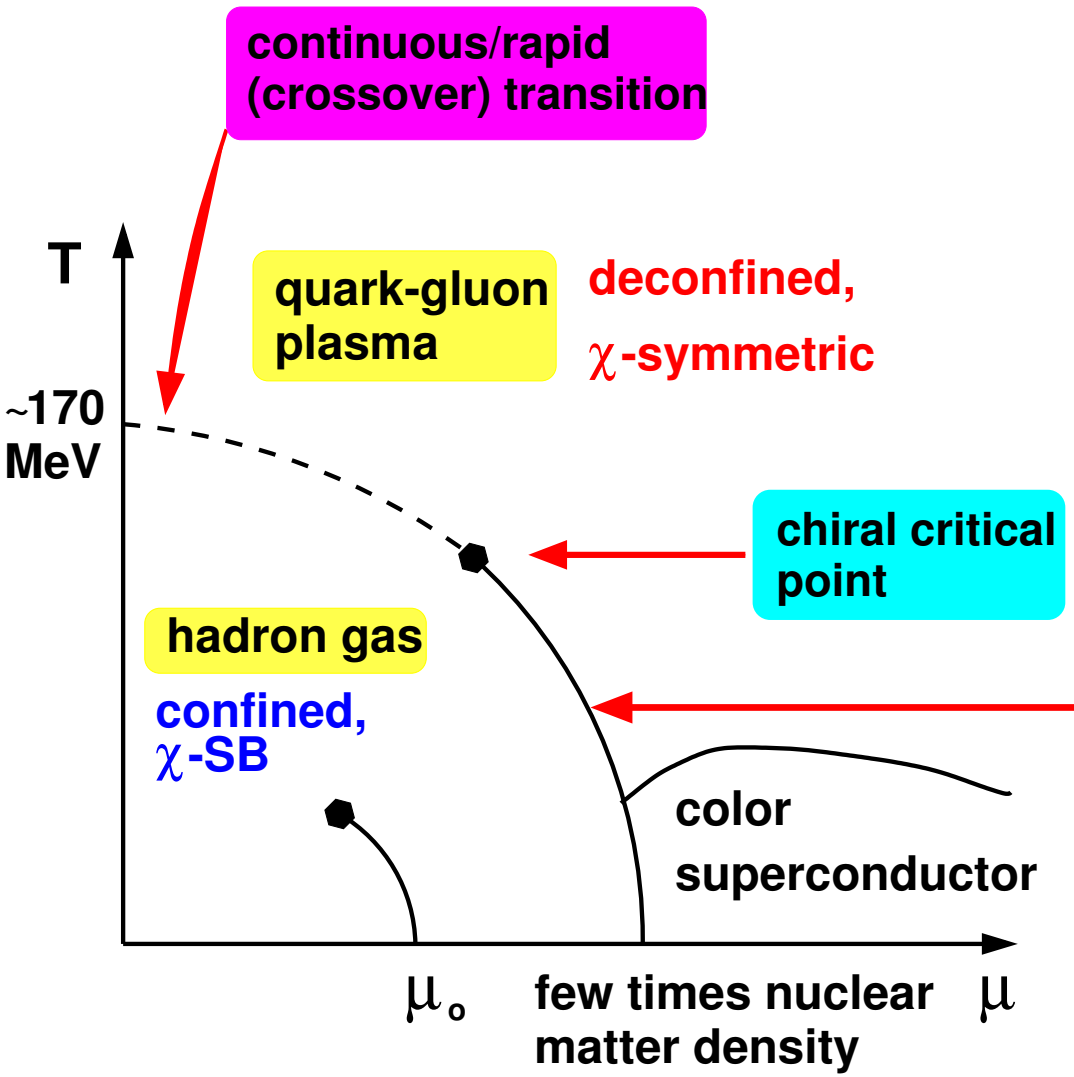

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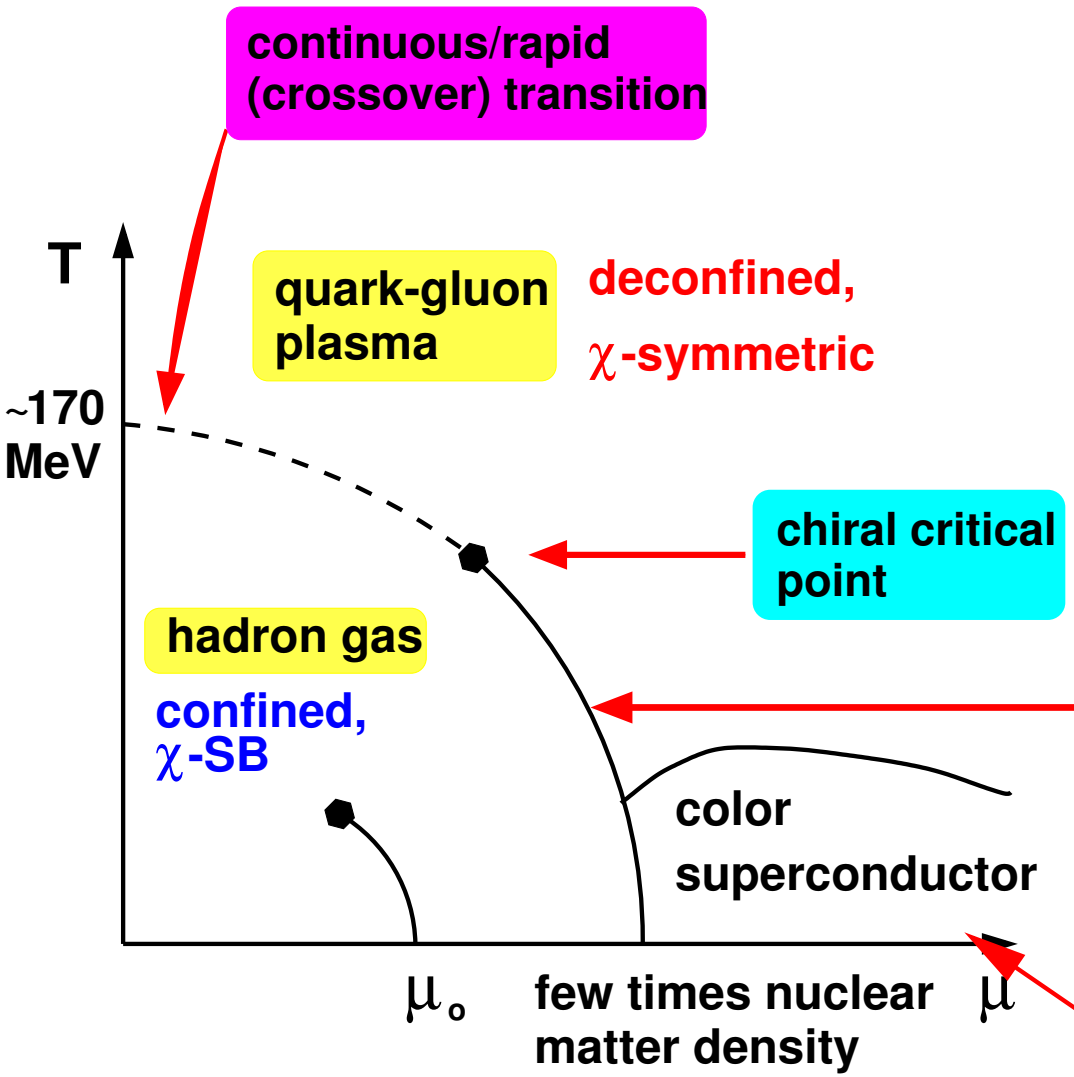

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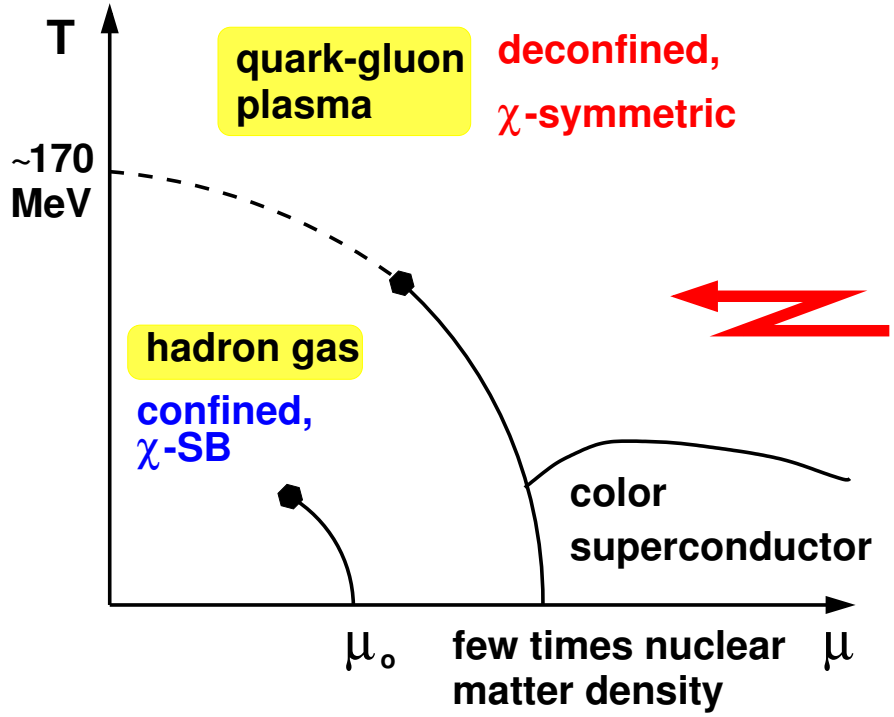
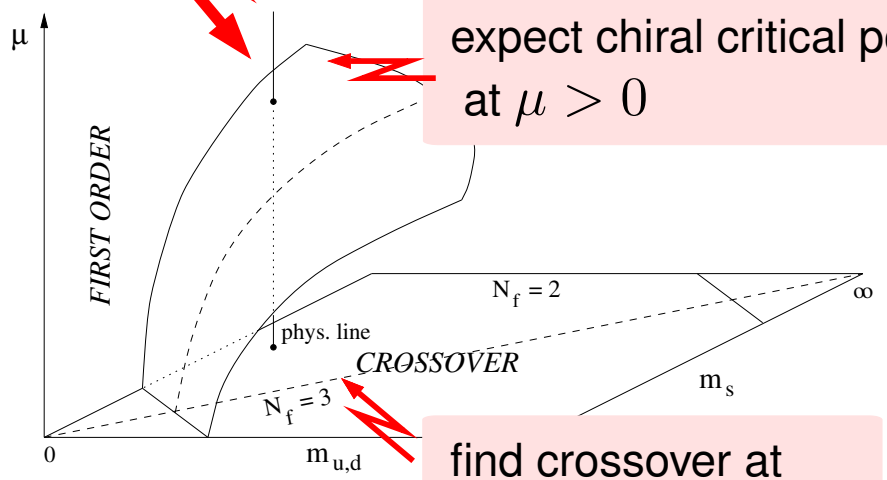
attractive 1–gluon exchange => qq–condensates

Quark mass dependence of the QCD phase diagram



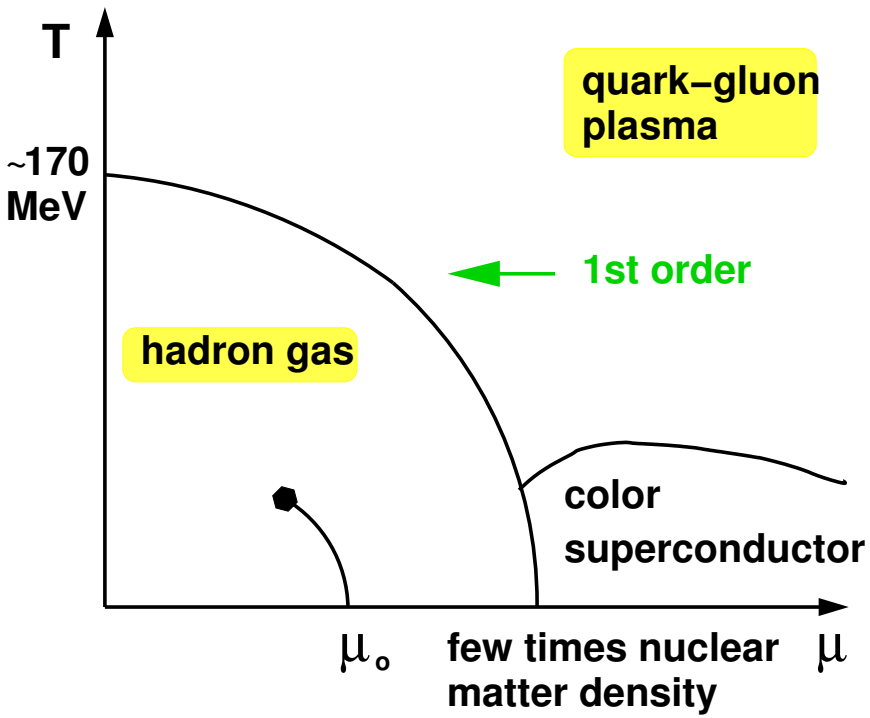
chiral critical point

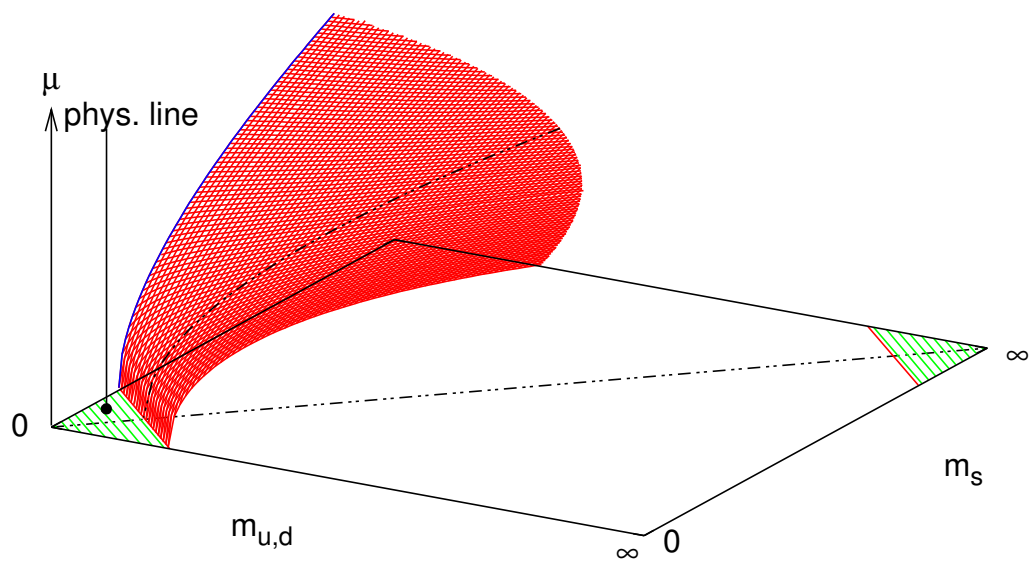


expect chiral critical point at $\mu > 0$

find crossover at $\mu = 0$

Quark mass dependence of the QCD phase diagram





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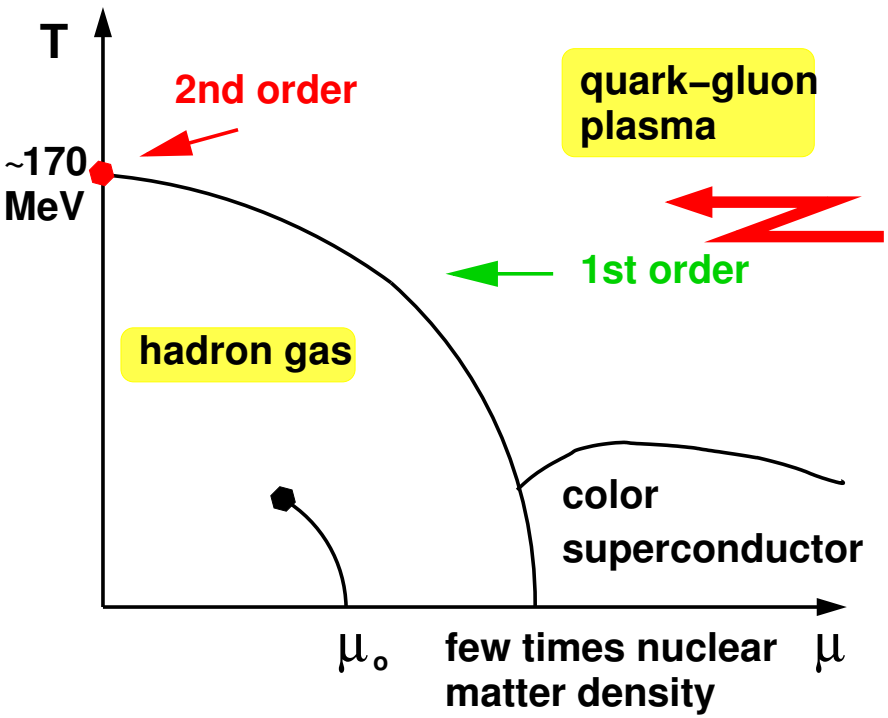

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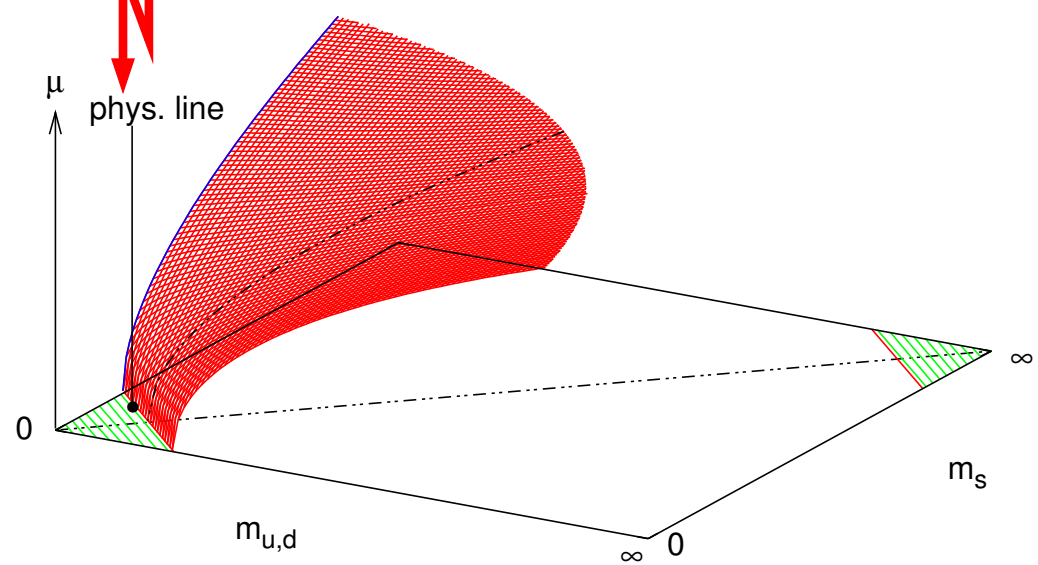

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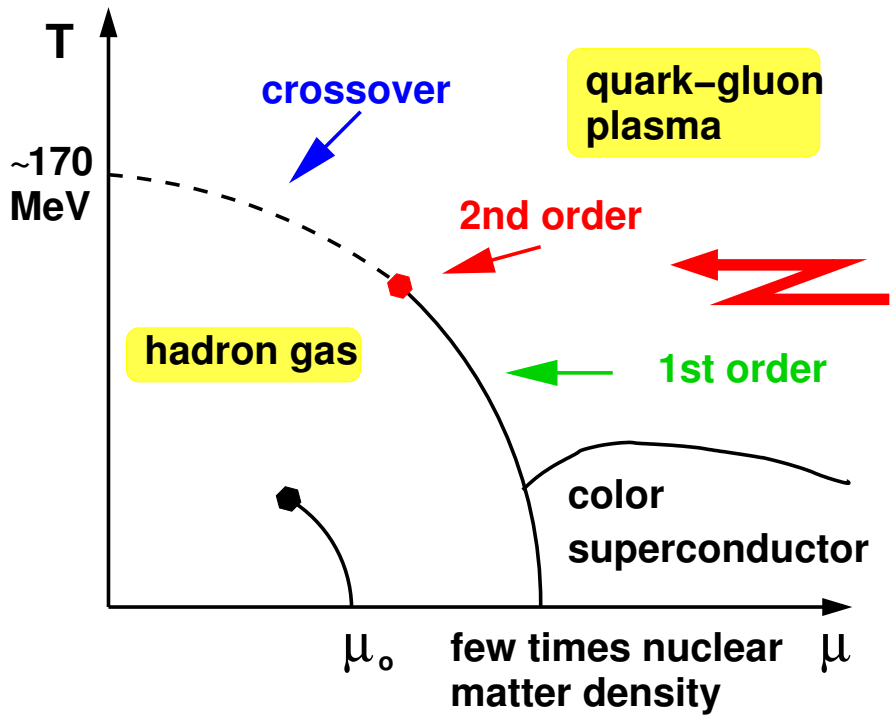

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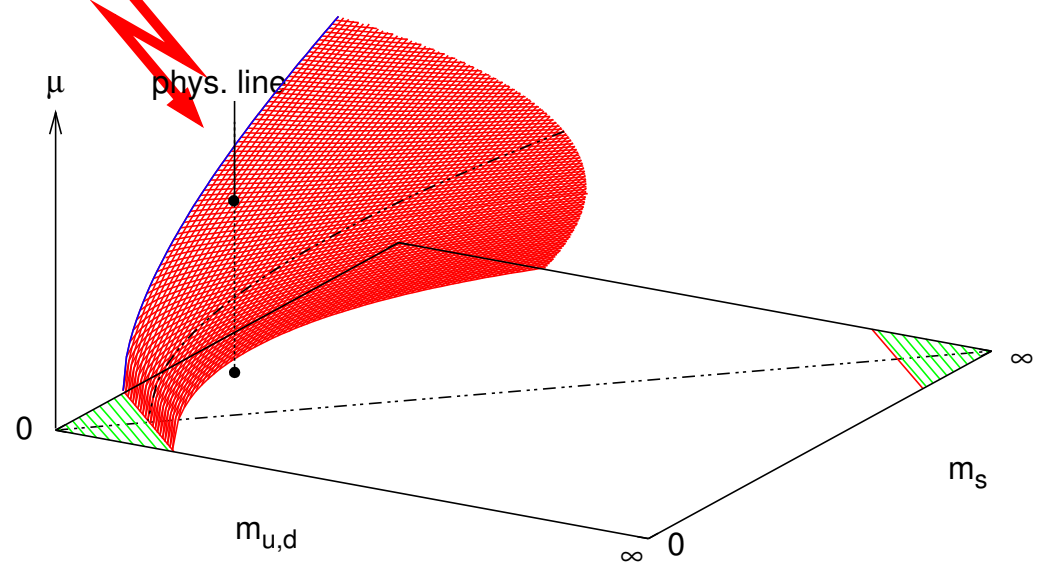

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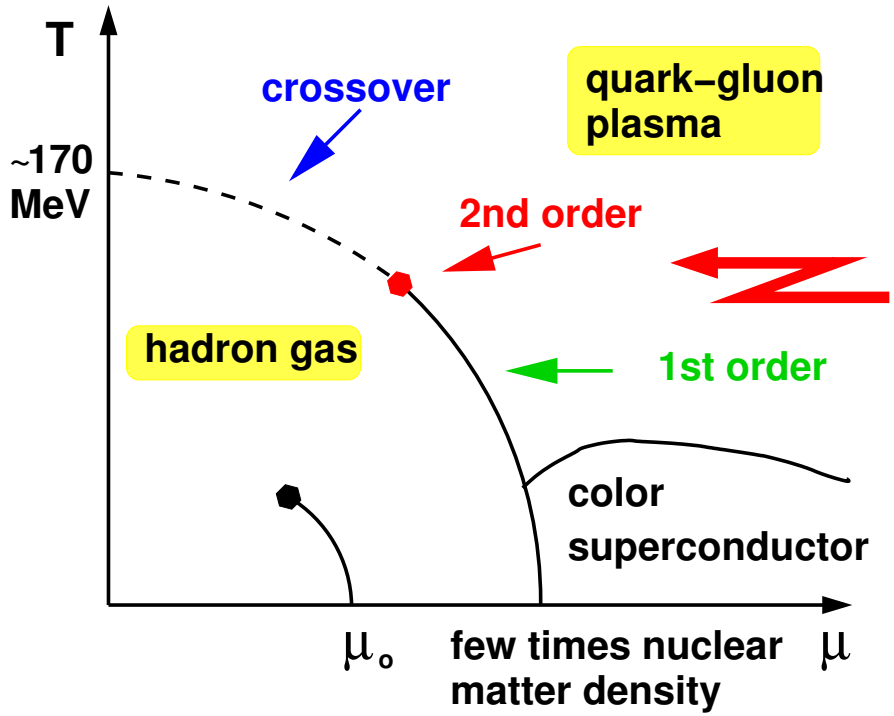

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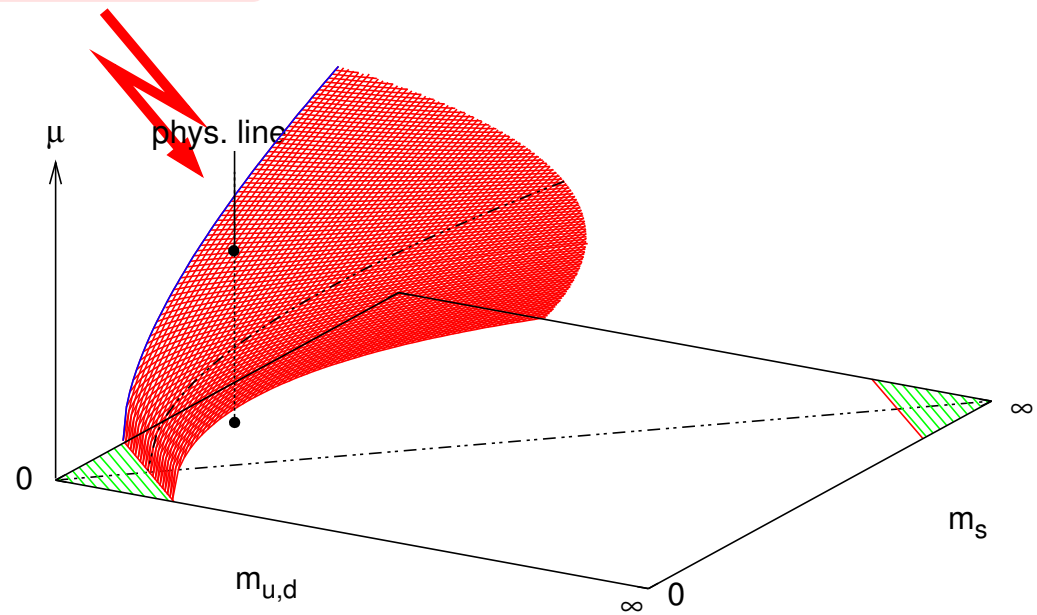
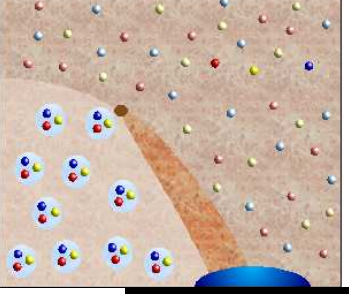
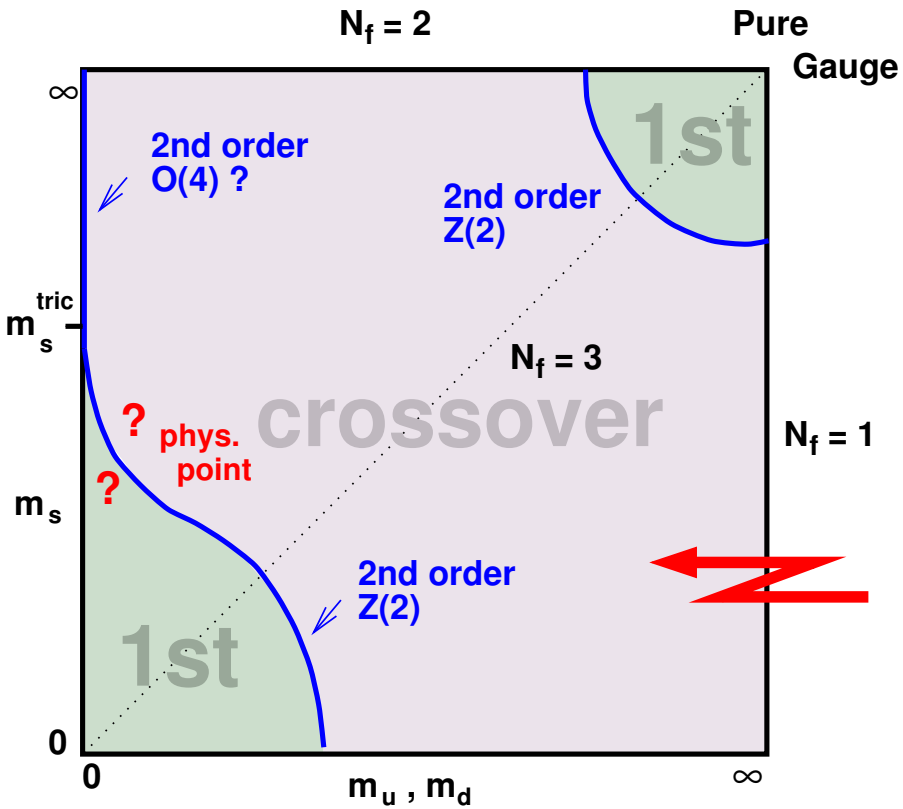


- locate critical line at $\mu = 0$



QCD phase diagram for vanishing chemical potential



- continuous transition for a wide range of quark masses

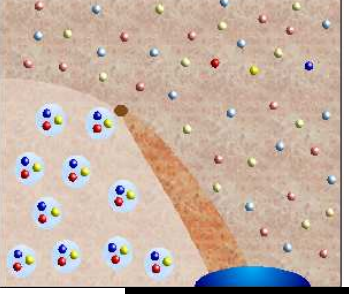
quark mass dependence

strong cut-off dependence

$m_{PS}^{crit} \simeq 290\ \mathrm{MeV}$, standard staggered

$m_{PS}^{crit} \simeq 70\ \mathrm{MeV}$, improved staggered

(Bielefeld, hep-lat/0309077)



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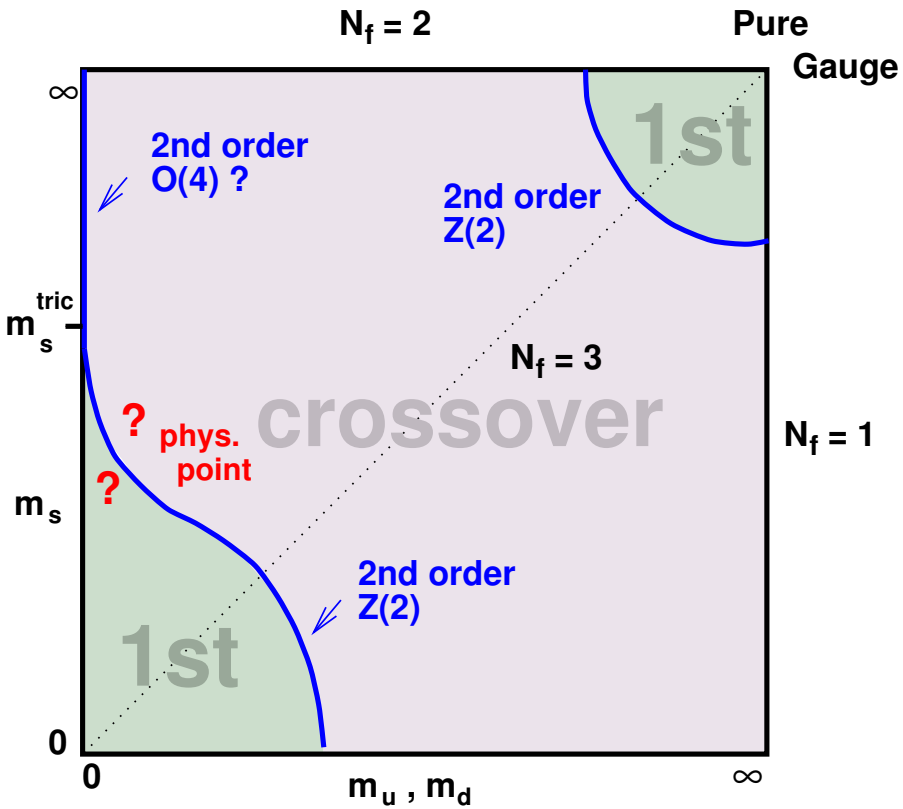


- continuous transition for a wide range of quark masses

$n_f = 3$:

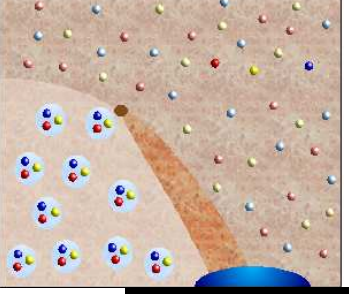
- critical point in Ising universality class

expansion around $m_s = m_{u,d}$:

$$m_s^{crit} = 3m_{u,d}^{crit,3f} - 2m_{u,d}$$

$$\Downarrow$$

$$\left(m_K^{crit}\right)^2 = \frac{3}{2}\left(m_\pi^{crit,3f}\right)^2 - \frac{1}{2}m_\pi^2$$



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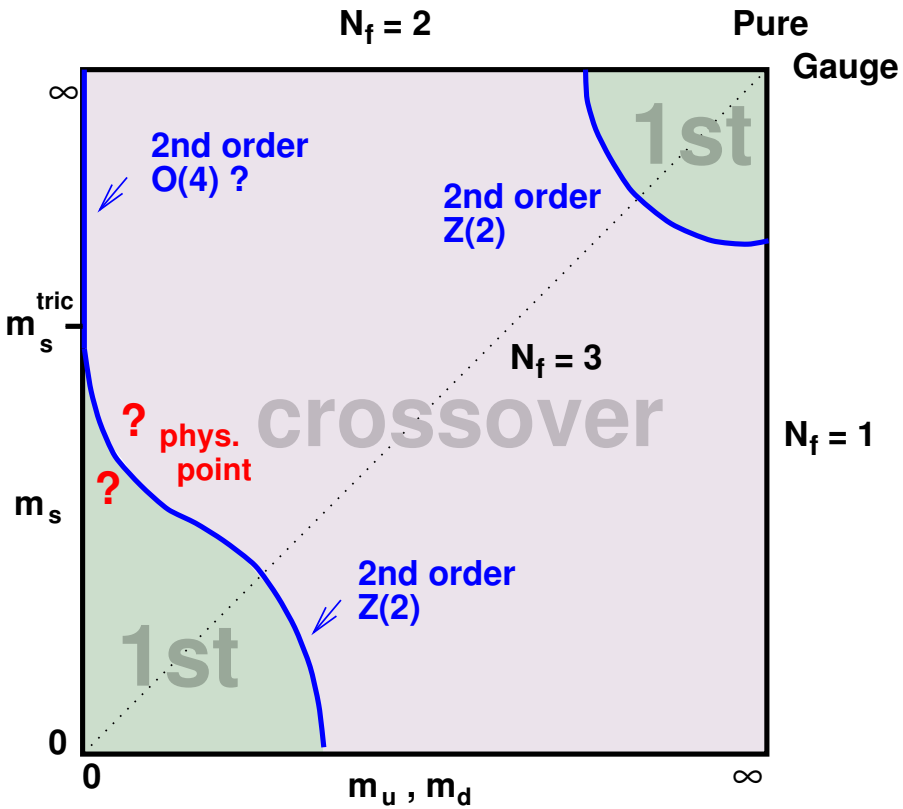
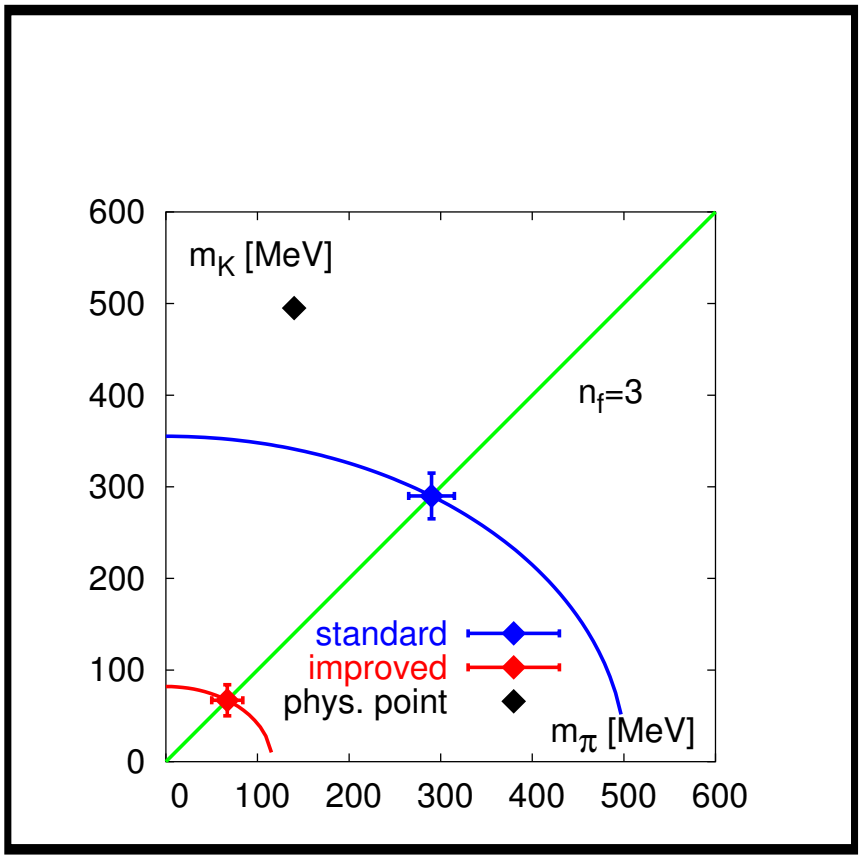
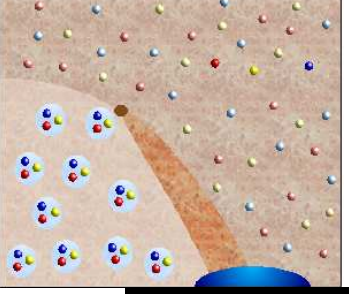


- continuous transition for a wide range of quark masses

- curvature of critical surface at $\mu = 0$ for $n_f = 3$ not crucial





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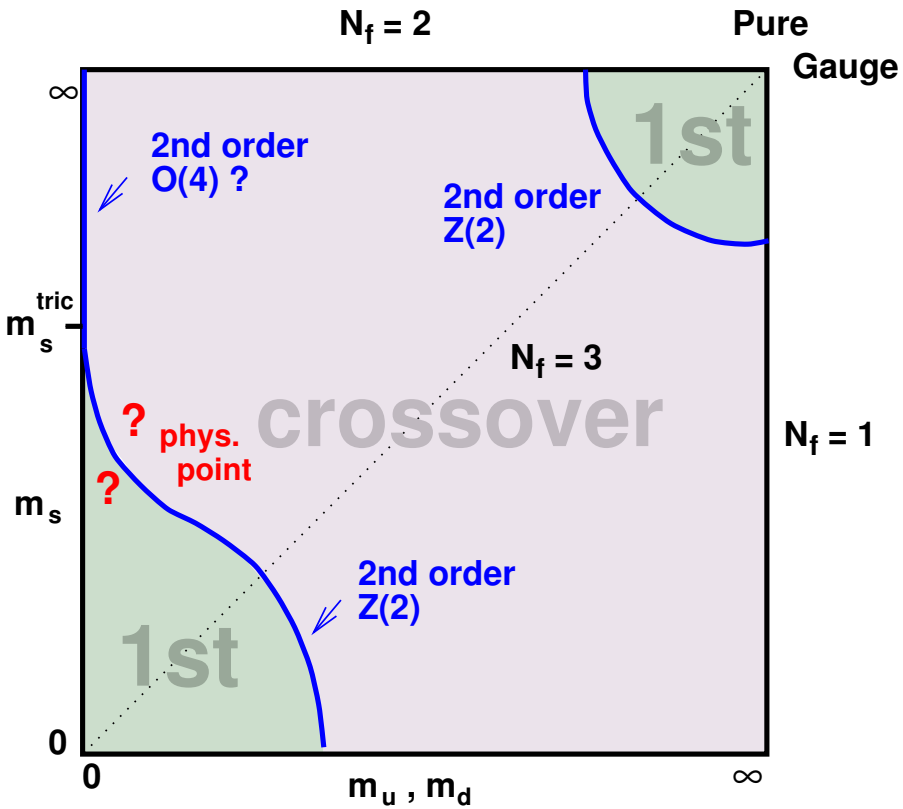

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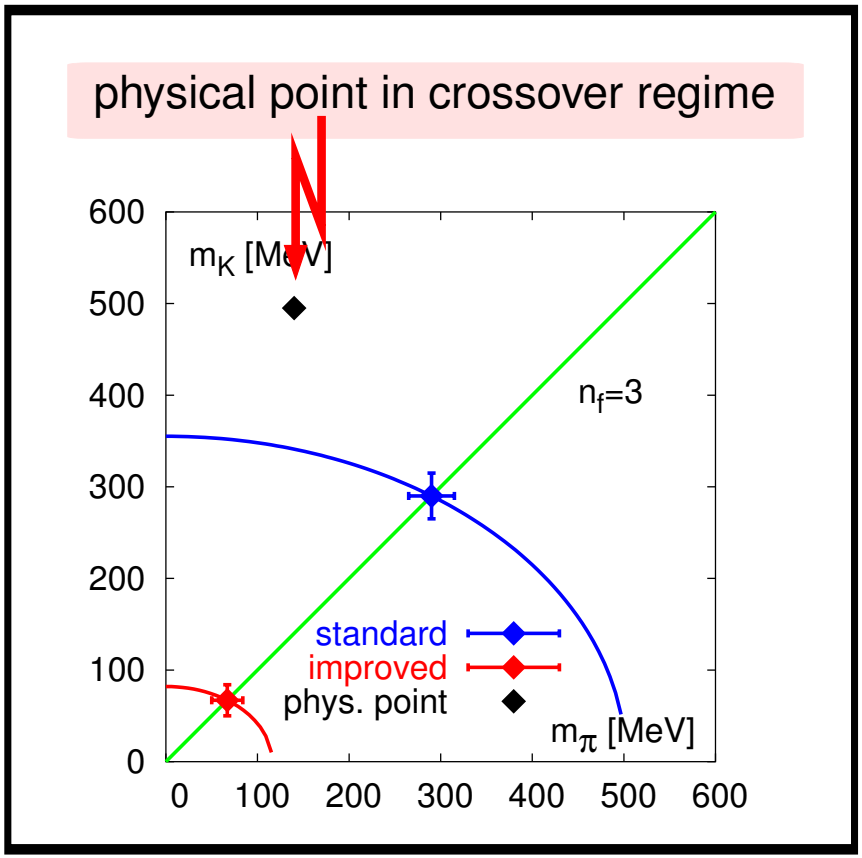
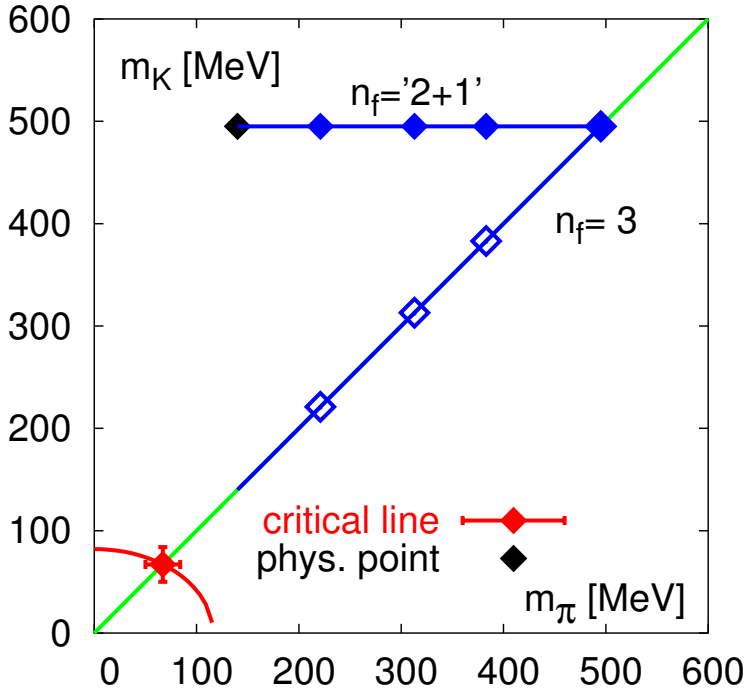


The chiral condensate: 2- vs. (2+1)- vs. 3-flavor QCD



- compare simulations in 3- and (2+1)-flavor QCD

C. Bernard et al. (MILC Col.), hep-lat/0405029

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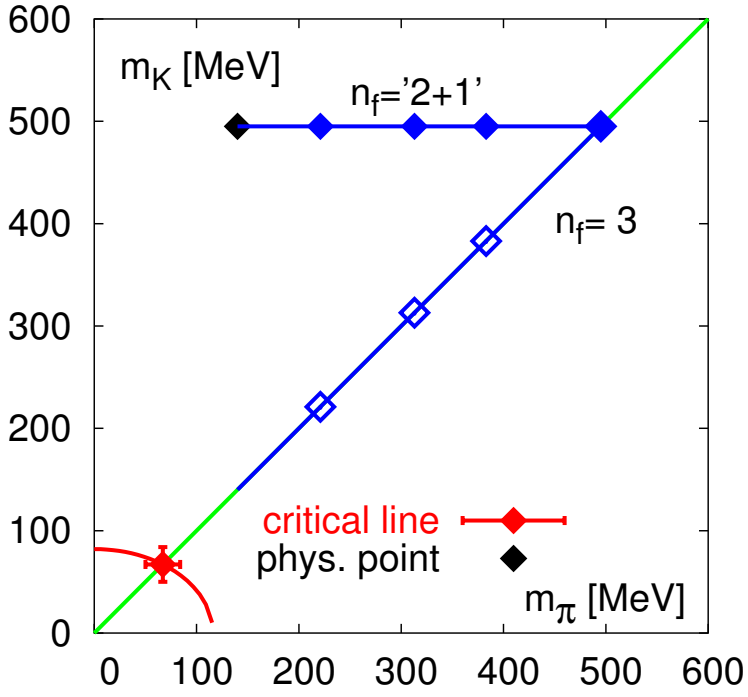
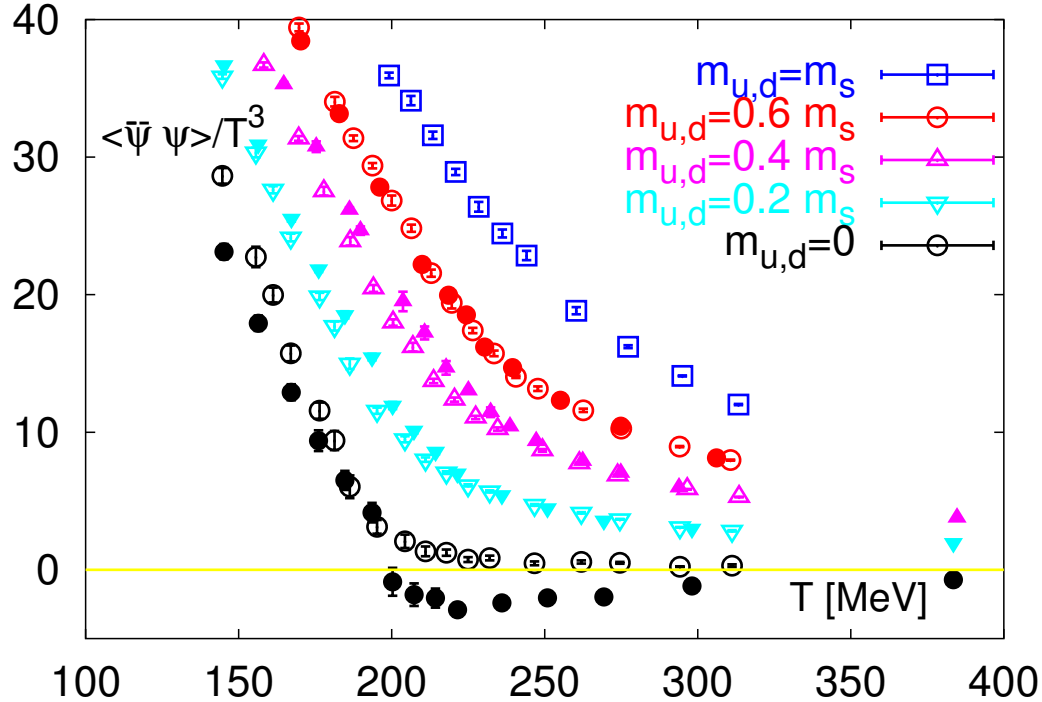




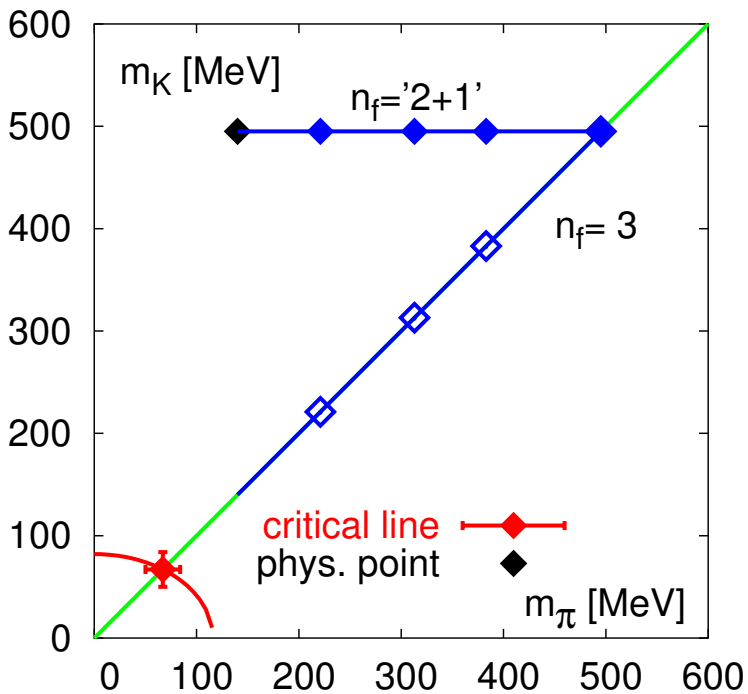
- compare simulations in 3- and (2+1)-flavor QCD

C. Bernard et al. (MILC Col.), hep-lat/0405029

- $\langle\bar\psi\psi\rangle$ for 3- and (2+1)-flavor QCD almost identical;
- sensitive only to light quark mass

$\Rightarrow$ chiral properties of 2-flavor QCD dominate (pseudo)-critical behavior

The chiral condensate: 2- vs. (2+1)- vs. 3-flavor QCD



$n_f = 2$:

- O(4) universality class expected, but: could not yet be verified

FK, E. Laermann, PRD50 (1994) 6954

doubts:

evidence for 1st order transition??

A. DiGiacomo et al., hep-lat/0408008

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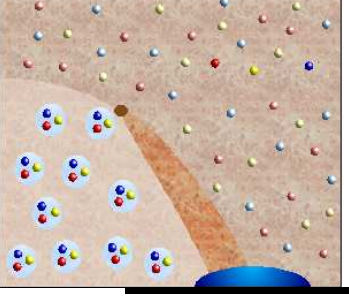


Extending the phase diagram to non-vanishing chemical potential

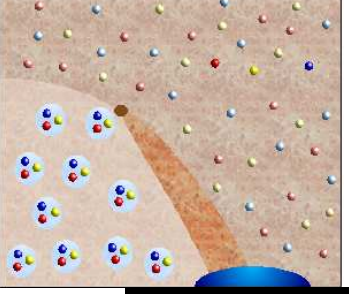
non-zero baryon number density: $\mu > 0$

$$Z(V,T,\mu) = \int \mathcal{D}\mathcal{A}\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \mathrm{e}^{-S_E(V,T,\mu)}$$
$$= \int \mathcal{D}\mathcal{A}\mathcal{D}\ det\ M(\mu)\ \mathrm{e}^{-S_E(V,T)}$$

$\Uparrow$complex fermion determinant;

long standing problem

$\Rightarrow$ three (partial) solutions for large T, small μ



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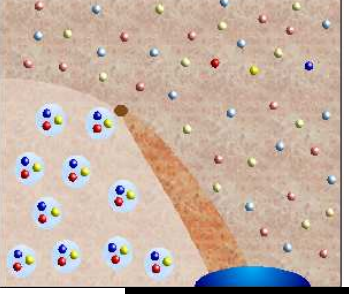
- exact evaluation of $detM$: works well on small lattices; requires reweighting
  Z. Fodor, S.D. Katz, JHEP 0203 (2002) 014
- Taylor expansion around $\mu = 0$ works well for small μ; has truncation errors
  C. R. Allton et al. (Bielefeld-Swansea), Phys. Rev. D66 (2002) 074507
- imaginary chemical potential: works well for small μ; requires analytic continuation
  Ph. deForcrand, O. Philipsen, Nucl. Phys. B642 (2002) 290



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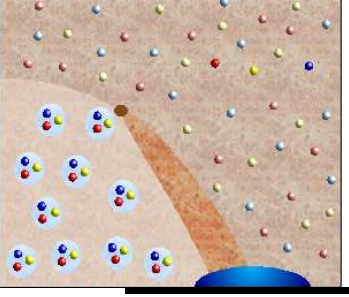
$$T_c(\mu) = ? \qquad \boxed{\frac{\mathrm{d}T_c}{\mathrm{d}(\mu)^2} = -\frac{1}{N_\tau^2 T_c(0)} \frac{\partial g_c^{-2}(\mu a)}{\partial(\mu a)^2}\left(a\frac{\partial g^{-2}(a)}{\partial a}\right)}$$

- determine μ-dependence of Lee-Yang zeroes of partition function
- determine shift of peak positions of susceptibilities and/or radius of convergence of Taylor series
- locate phase transition (crossover) for imaginary μ



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pert. or non-pert. β-function

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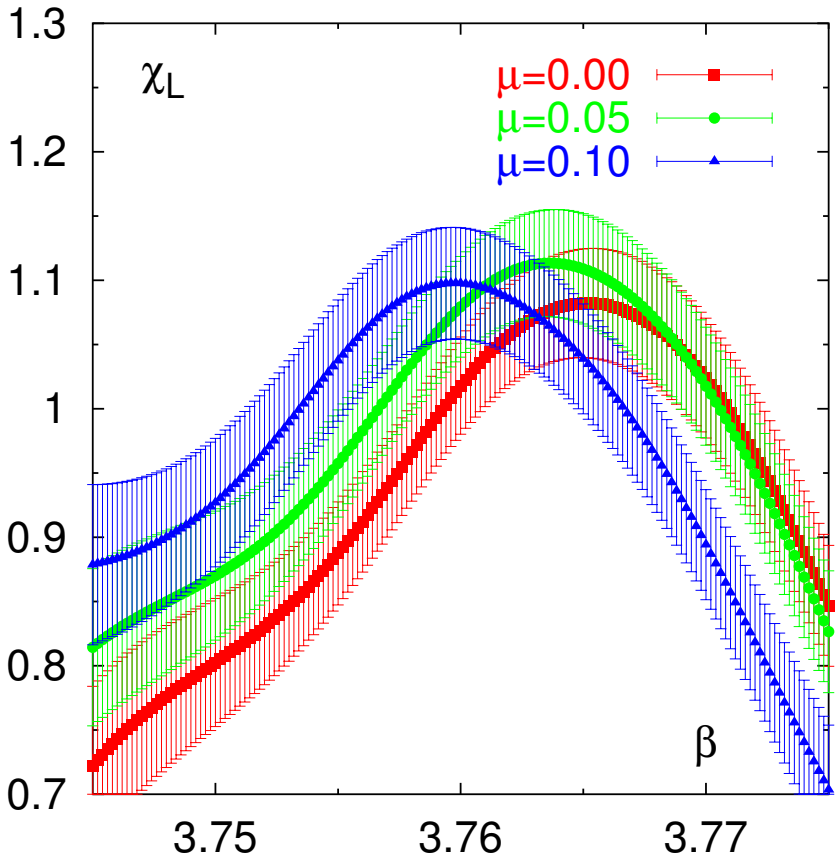
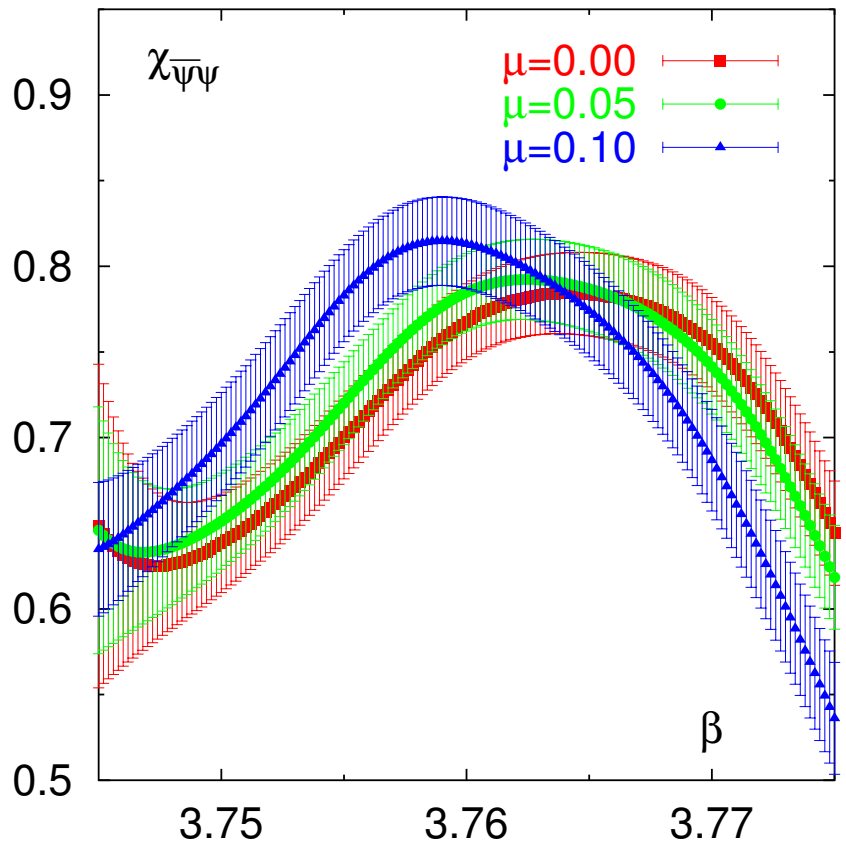
Reweighting of susceptibilities for small μ

Polyakov loop susceptibility



chiral susceptibility

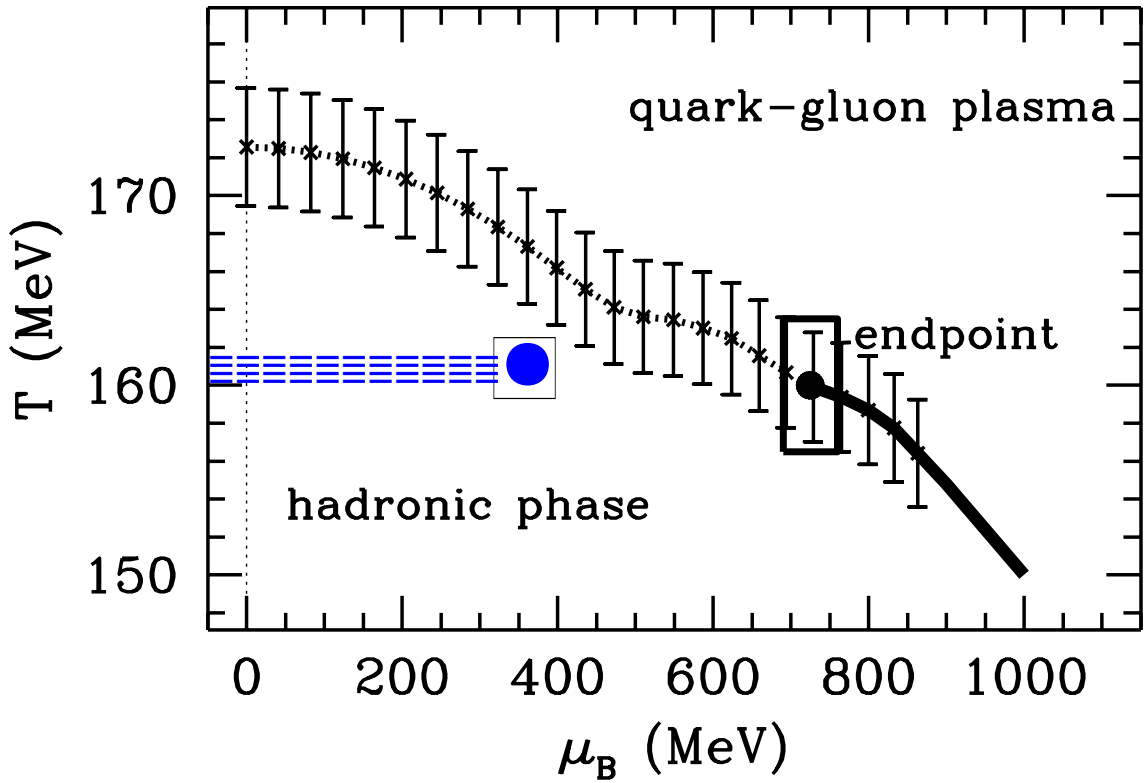


$$T_c(\mu_B)/T_c(0) = 1 - 0.0078(38)\,(\mu_B/T_c(0))^2$$ (non-pert.)

C. R. Allton et al. (Bielefeld-Swansea), Phys. Rev. D66 (2002) 074507

Extending the phase diagram to non-vanishing chemical potential

analysis of volume dependence of Lee-Yang zeroes for $\mu > 0$



Fodor & Katz,
JHEP 0203 (2002) 014
$V = 4^3,\ 6^3, 8^3$

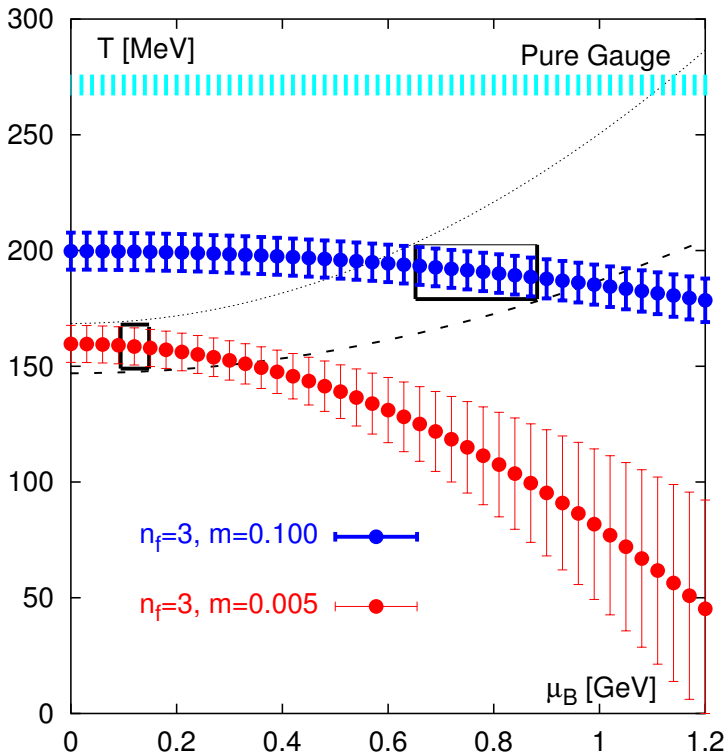
Fodor & Katz,
JHEP 0404 (2004) 050
$V = 6^3,\ 8^3, 10^3, 12^3$

$$\mu^{crit} = 360(40)\ \mathrm{MeV}$$

$$T_c(\mu_B)/T_c(0) = 1 - 0.0032(1)\,(\mu_B/T_c(0))^2 \qquad \text{(pert.)}$$

Extending the phase diagram to non-vanishing chemical potential

first (exploratory) results on the quark mass dependence of the transition line:



m_q-dependence

(3-flavor QCD, pert. β-function, Taylor expansion)

$$\frac{T_c(\mu)}{T_c(0)} \; : \; 1 - 0.025(6)(\mu_q/T)^2 \; , \; ma = 0.1$$

$$1 - 0.114(46)(\mu_q/T)^2 \; , \; ma = 0.005$$

Bielefeld-Swansea
(hep-lat/0309116, Lattice 2003)

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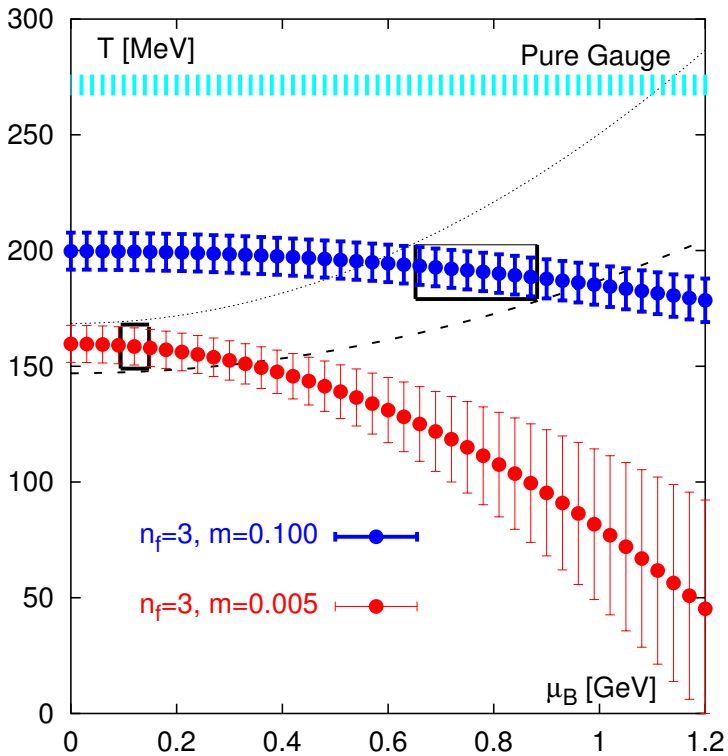

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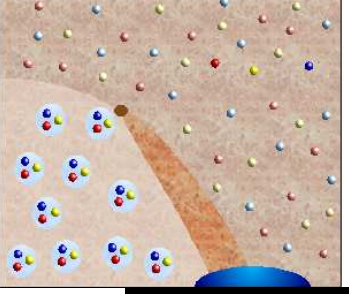
Bielefeld-Swansea
(hep-lat/0309116, Lattice 2003)

a systematic analysis of
cut-off effects, scaling violations
AND volume + truncation effects
still needs to be done

m_q-dependence not confirmed in simulations with imaginary μ
Ph. de Forcrand, O. Philipsen, NP B673 (2003) 170



Extending the phase diagram to non-vanishing chemical potential

non-zero baryon number density: $\mu > 0$

$$Z(V,T,\mu) = \int \mathcal{D}\mathcal{A}\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \mathrm{e}^{-S_E(V,T,\mu)}$$
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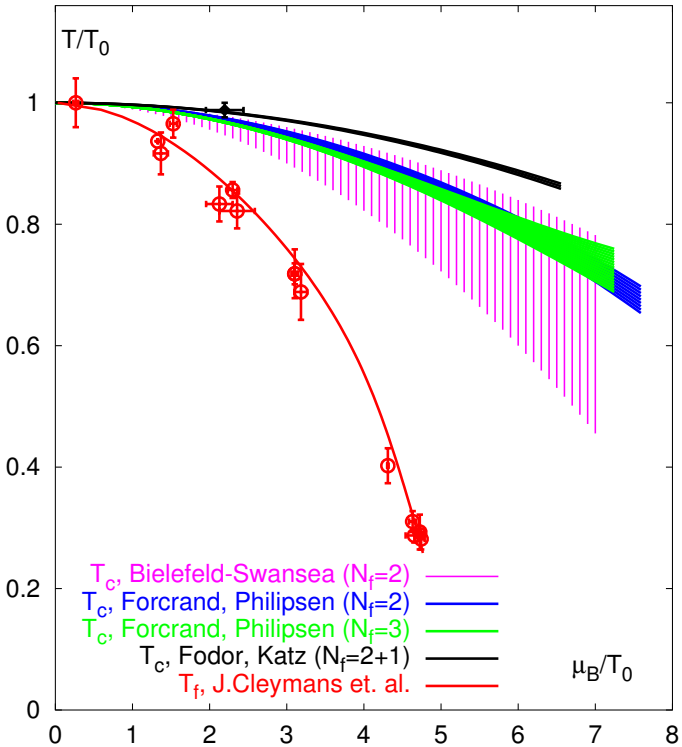

$\frac{T_c(\mu)}{T_c(0)}$: $1 - 0.0056(4)(\mu_B/T)^2$

deForcrand, Philipsen (imag. μ, pert)

$1 - 0.0078(38)(\mu_B/T)^2$

Bielefeld-Swansea

($\mathcal{O}(\mu^2)$ reweighting, non-pert)

$1 - 0.0032(1)(\mu_B/T)^2$

Fodor,Katz(Lee-Yang zeroes, pert)

Extending the phase diagram to non-vanishing chemical potential

non-zero baryon number density: $\mu > 0$

$$Z(V,T,\mu) = \int \mathcal{D}\mathcal{A}\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \mathrm{e}^{-S_E(V,T,\mu)}$$
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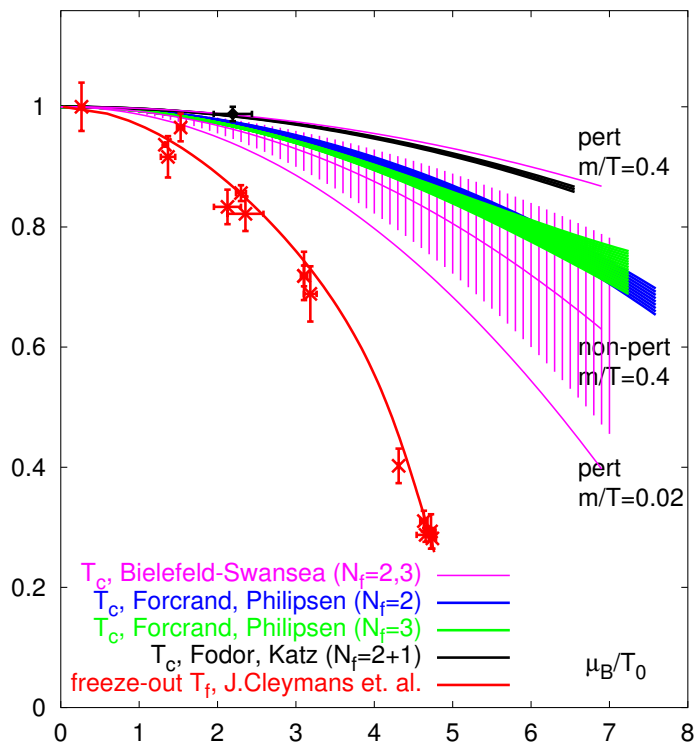
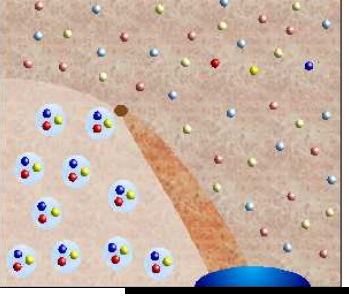


current studies of $T_c(\mu)$ are exploratory!

uncertainties in scale-determination and

systematics of quark mass dependencee



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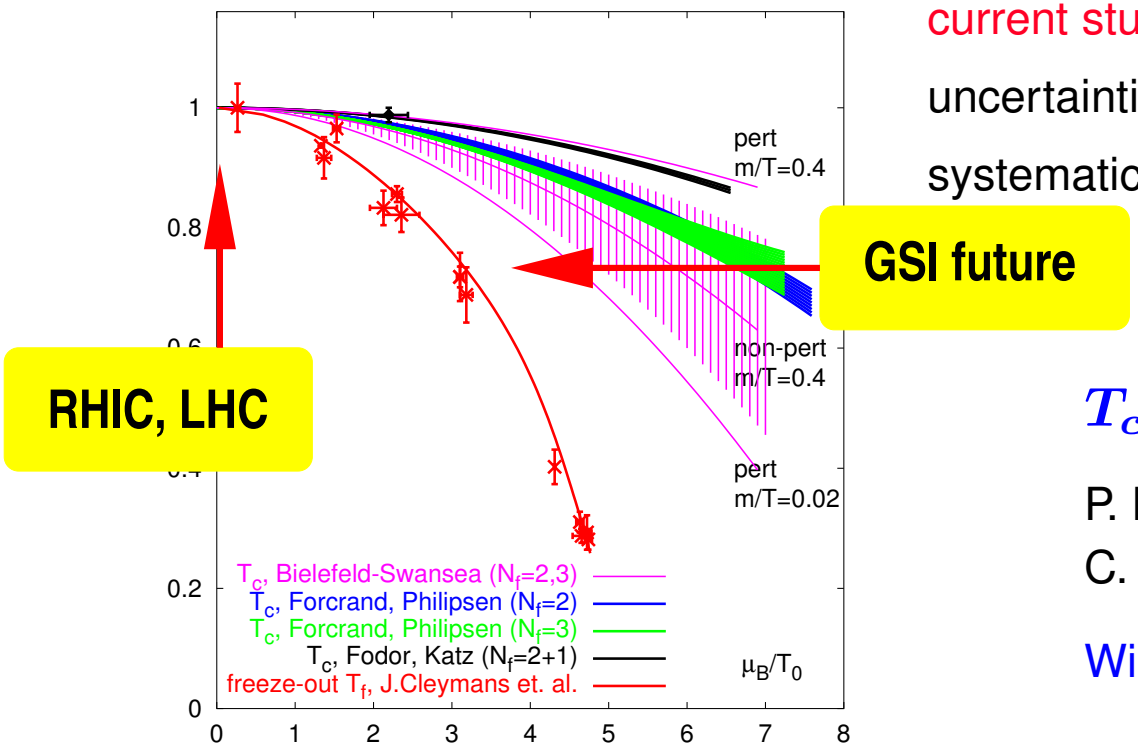


current studies of $T_c(\mu)$ are exploratory!

uncertainties in scale-determination and

systematics of quark mass dependencee

$T_c(\mu) \equiv T_{\mathrm{freeze}}$?

P. Braun-Munzinger, J. Stachel,
C. Wetterich, hep-nucl/0311005

Will be answered by LGT calculations

Intermezzo: Next generation computers for lattice gauge theory

120 GFlops APEmille in Bielefeld

today:

computing resources not sufficient
for reliable thermodynamics calculations
with physical quark mass parameters,
$\mu \neq 0$ and large enough statistics
on large enough lattices

Intermezzo: Next generation computers for lattice gauge theory

QCDOC and apeNEXT

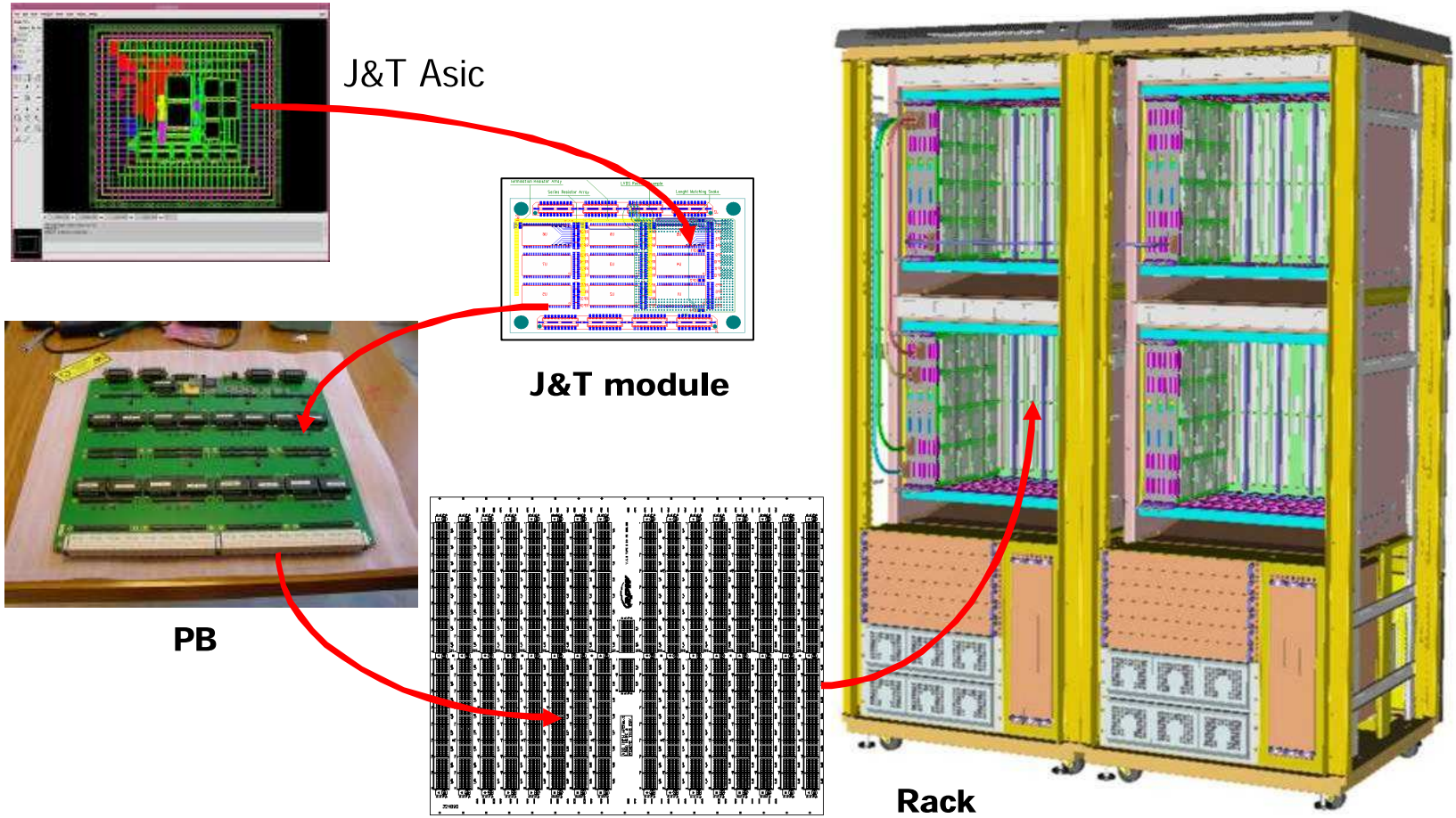
2004/05:

QCD thermodynamics on the next generation of special purpose dedicated QCD computers

installations with (10-20) TFlops peak speed are planned in the USA and Europe

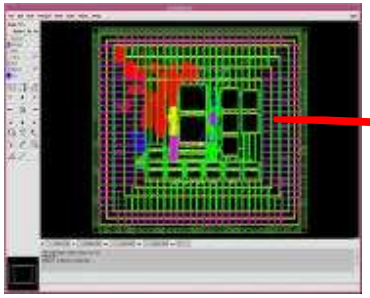
apeNEXT: Next generation of APE computers

Assembling apeNEXT...

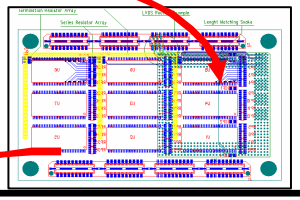


apeNEXT: Next generation of APE computers

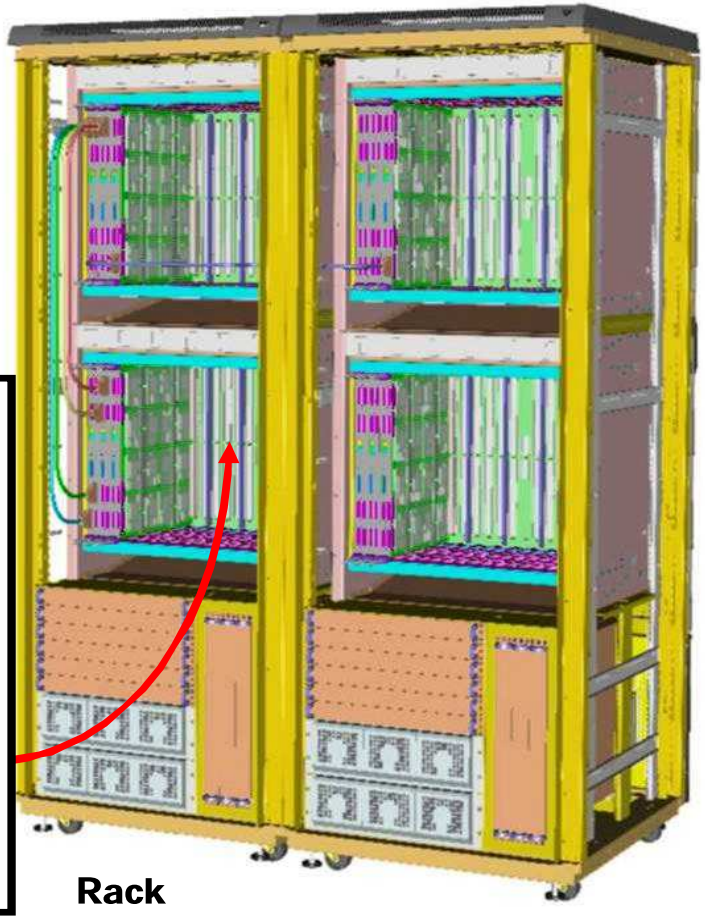
Assembling apeNEXT...



J&T Asic



12/2003

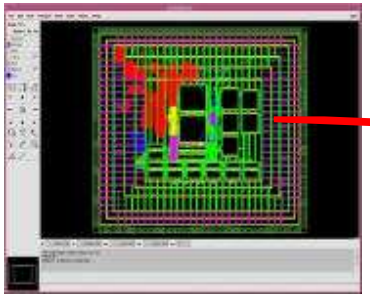


Rack

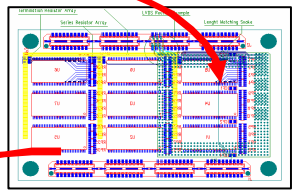
BackPlane

apeNEXT: Next generation of APE computers

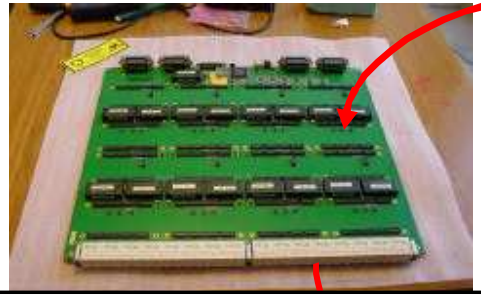
Assembling apeNEXT...

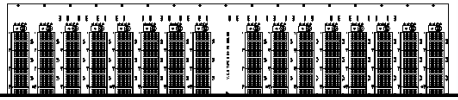


J&T Asic

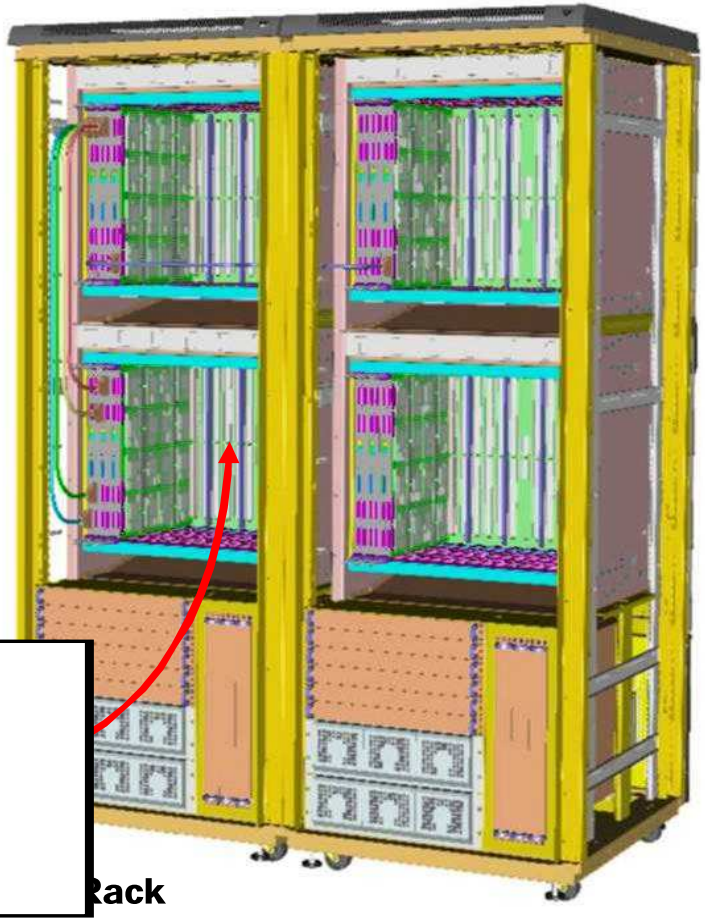


J&T module





BackPlane



Rack

- first chips Dec. 2003
- two 0.8 TFlops prototypes $\sim$ autumn 2004
- first 3 TFlops installations in 2005

QCDOC: Next generation of Columbia-RIKEN computer

Columbia – RIKEN – UKQCD Collaboration

512 – node machine : (360 – 450) GFlops

- currently debugging three 0.9-Teraflops maschines (09/2004):
  1024-node systems, 0.5 Tbyte memory; 6 Gbit/sec Ethernet I/O bandwidth

QCDOC computing center at BNL :

- 10 TFlops machine for RBRC: $\sim$ autumn 2004
- 10 TFlops machine for american LGT community: $\sim$ early 2005
- ... larger installations possible and needed!

Bulk thermodynamics with non-vanishing chemical potential

$$Z(V,T,\mu) = \int \mathcal{D}\mathcal{A}\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \mathrm{e}^{-S_E(V,T,\mu)}$$
$$= \int \mathcal{D}\mathcal{A}\ \left[det\ M(\mu)\right]^f\ \mathrm{e}^{-S_G(V,T)}$$

$\Uparrow$complex fermion determinant;

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$\Uparrow$complex fermion determinant;

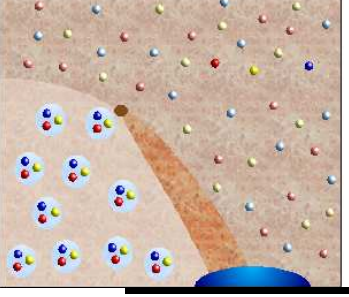
$\Downarrow$Taylor expansion;

$$\frac{p}{T^4} = \frac{1}{VT^3}\ln Z(V,T,\mu)$$

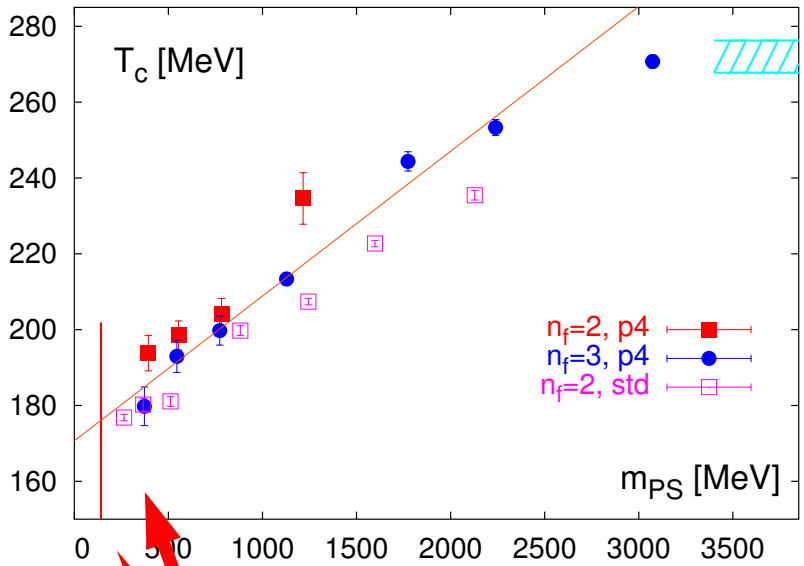
$$\equiv \sum_{n=0}^{\infty} c_n(T)\left(\frac{\mu}{T}\right)^n$$

$$= c_0 + c_2\left(\frac{\mu}{T}\right)^2 + c_4\left(\frac{\mu}{T}\right)^4 + \mathcal{O}((\mu/T)^6)$$

$$\mu = 0 \quad \Rightarrow \quad \frac{p}{T^4} \equiv c_0(T)$$

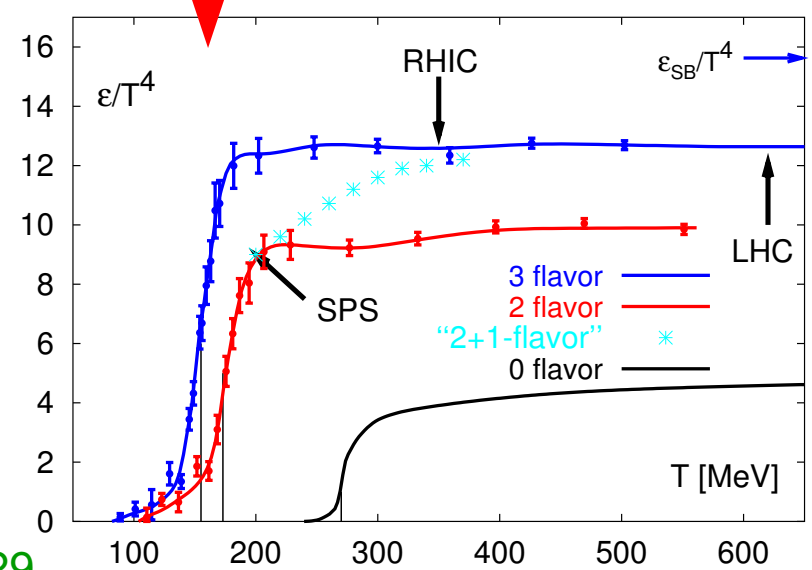


$\mu \equiv 0$: T_c and equation of state



$$\epsilon_c \simeq (6 \pm 2)\, T_c^4$$
$$\simeq (0.3 - 1.3)\, GeV/fm^3$$

energy density for 0, 2 and 3-flavor QCD



$$T_c = (173 \pm 8 \pm sys)\, MeV$$

FK, E. Laermann, A. Peikert,
Nucl. Phys. B605 (2001) 579

$$T_c = 167(13)\, [177(11)]\, MeV$$

C. Bernard et al. (MILC Col.), hep-lat/0405029

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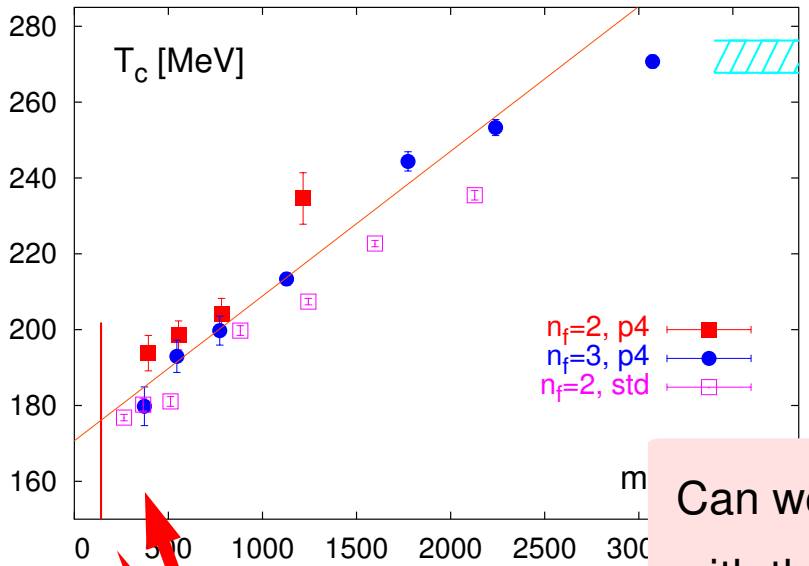

$$\epsilon_c \simeq (6 \pm 2)\, T_c^4$$
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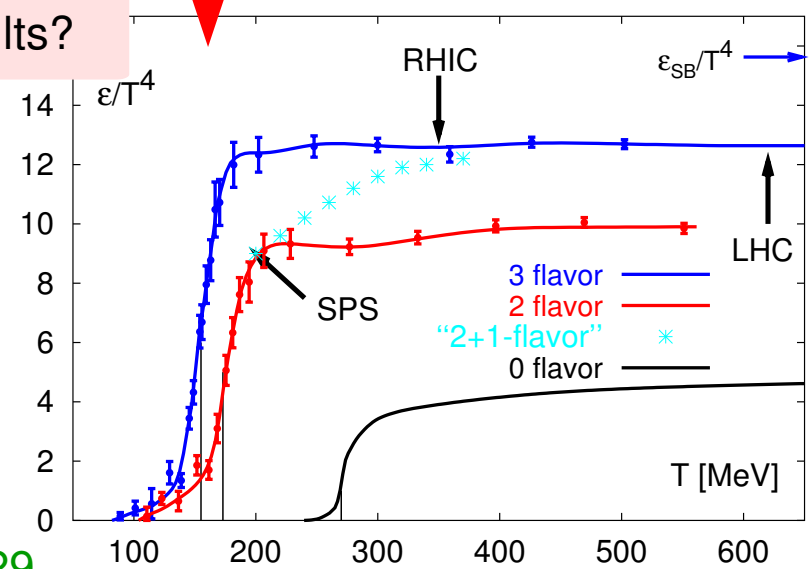
Can we be satisfied with these results?

gy density for 0, 2 and 3-flavor QCD



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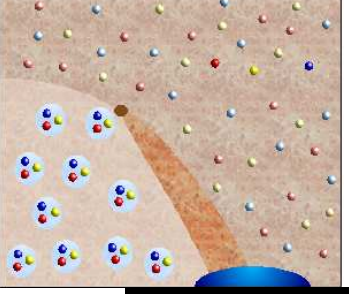

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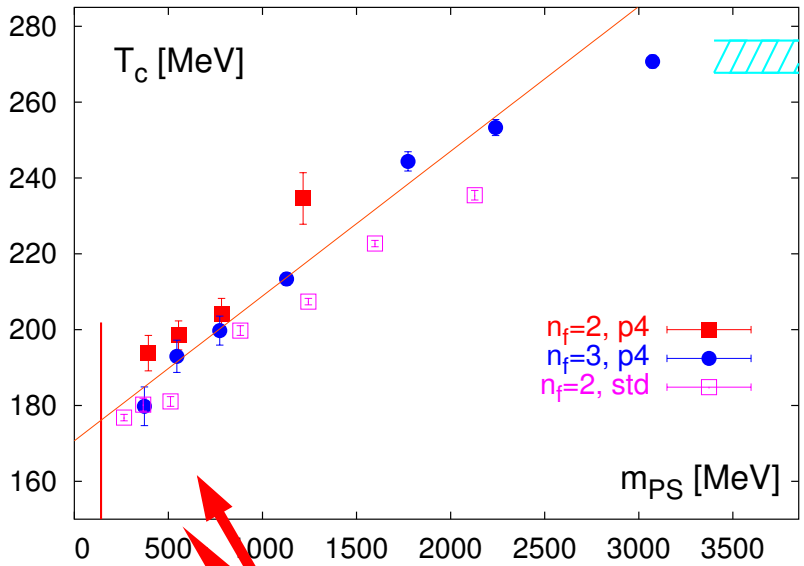


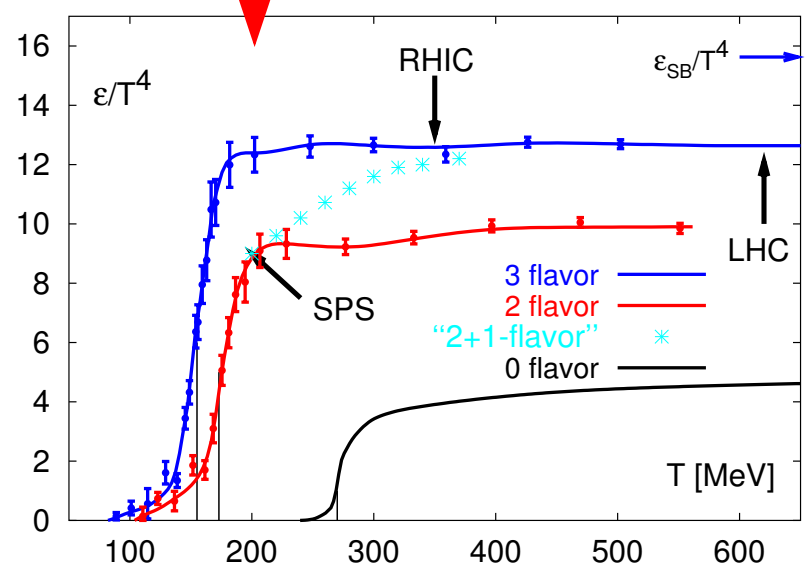
T_c

- $m_{PS} \gtrsim 300\ MeV$ (chiral limit??)
- $a \simeq 0.2\ fm$ (continuum limit??)
- improved staggered fermions,
  $\Rightarrow$ flavor symmetry breaking
  (need even better fermion actions)

ϵ_c

- $m_{PS} \simeq 770\ MeV$ (!!!)
- $V \simeq (4\ \text{fm})^3$ (thermodynamic limit)

energy density for 0, 2 and 3-flavor QCD



Pressure etc. up to $\mathcal{O}((\mu_q/T)^6)$

- Taylor expansion up to $\mathcal{O}\left((\mu_q/T)^6\right)$

$$\frac{p}{T^4} = \frac{1}{VT^3}\ln\mathcal{Z} = \sum_{n=0}^{\infty} c_n(T)\left(\frac{\mu_q}{T}\right)^n \simeq c_0 + c_2\left(\frac{\mu_q}{T}\right)^2 + c_4\left(\frac{\mu_q}{T}\right)^4 + c_6\left(\frac{\mu_q}{T}\right)^6$$

quark number density $\quad \frac{n_q}{T^3} = 2c_2\frac{\mu_q}{T} + 4c_4\left(\frac{\mu_q}{T}\right)^3 + 6c_6\left(\frac{\mu_q}{T}\right)^5$

quark number susceptibility $\quad \frac{\chi_q}{T^2} = 2c_2 + 12c_4\left(\frac{\mu_q}{T}\right)^2 + 30c_6\left(\frac{\mu_q}{T}\right)^4$

an estimator for the radius of convergence

$$\left(\frac{\mu_q}{T}\right)_{crit} = \lim_{n\to\infty}\left|\frac{c_{n-2}}{c_n}\right|^{1/2}$$

$c_n > 0$ for all n;
singularity for real μ

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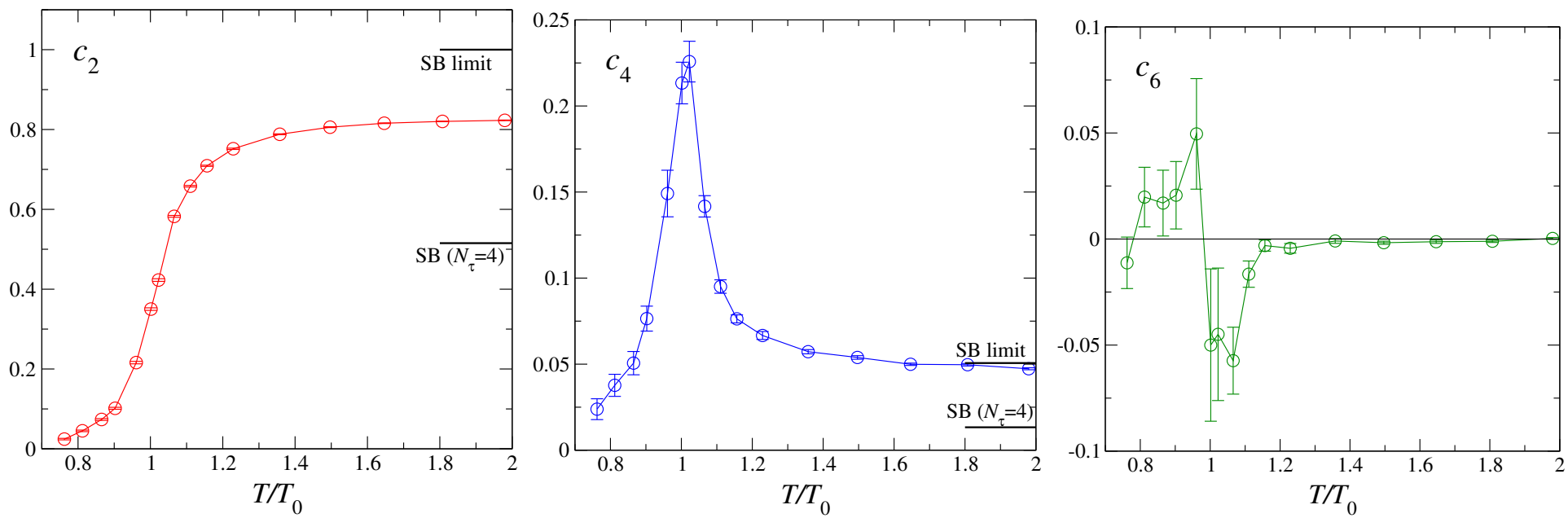

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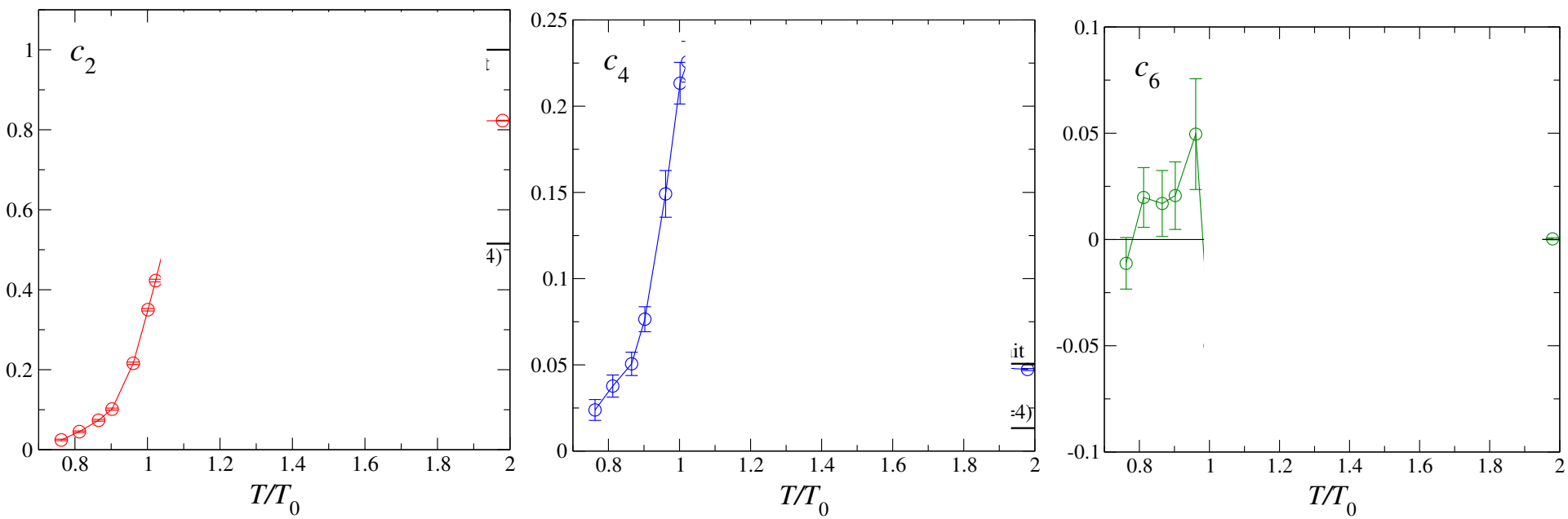

$c_n > 0$ for all n and $T \lesssim 0.95\, T_c$ $\Leftrightarrow$ singularity for real μ (positive μ^2)

Pressure etc. up to $\mathcal{O}((\mu_q/T)^6)$

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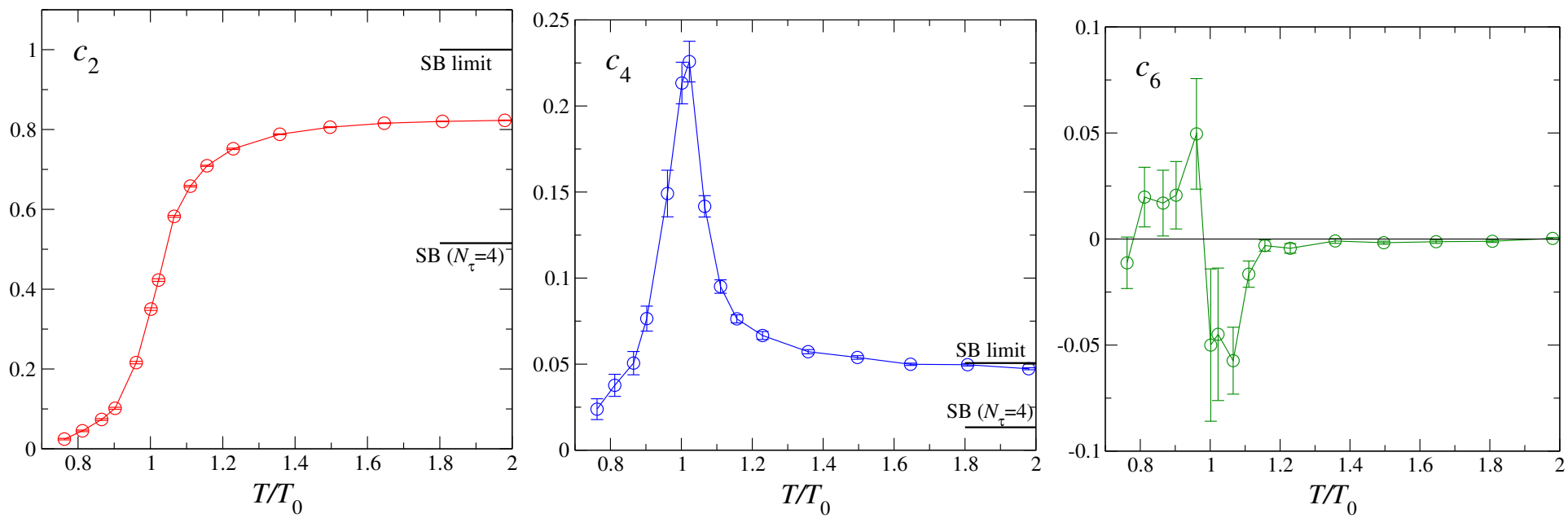
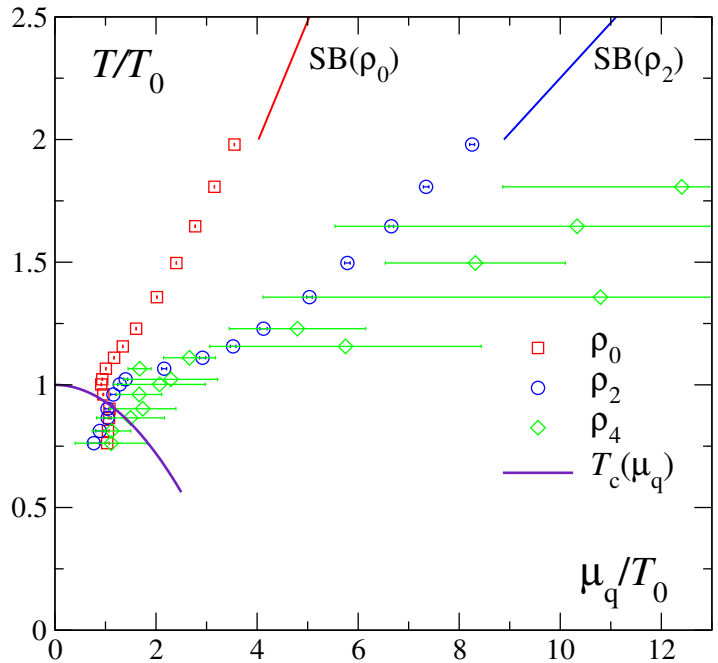
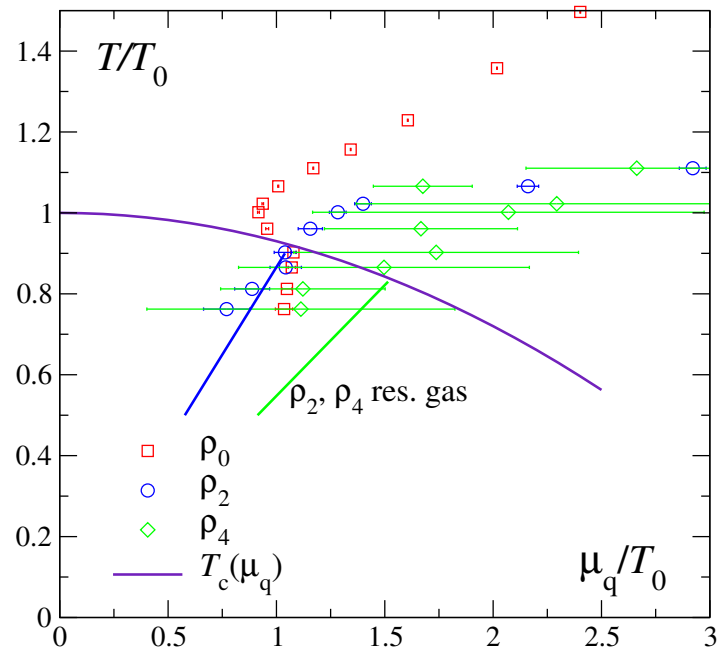


irregular sign of c_n for $T \gtrsim T_c$ $\Leftrightarrow$ singularity in complex plane

Radius of convergence: lattice estimates vs. resonance gas

- Taylor expansion $\Rightarrow$ estimates for radius of convergence $\rho_n = \sqrt{\left|\frac{c_{2n-2}}{c_{2n}}\right|}$





$T < T_0$: $0.8 \lesssim \rho_n \lesssim 1.5$ for all n $\Rightarrow$ $400\ \mathrm{MeV} \lesssim \mu_B^{crit} \lesssim 800\ \mathrm{MeV}$

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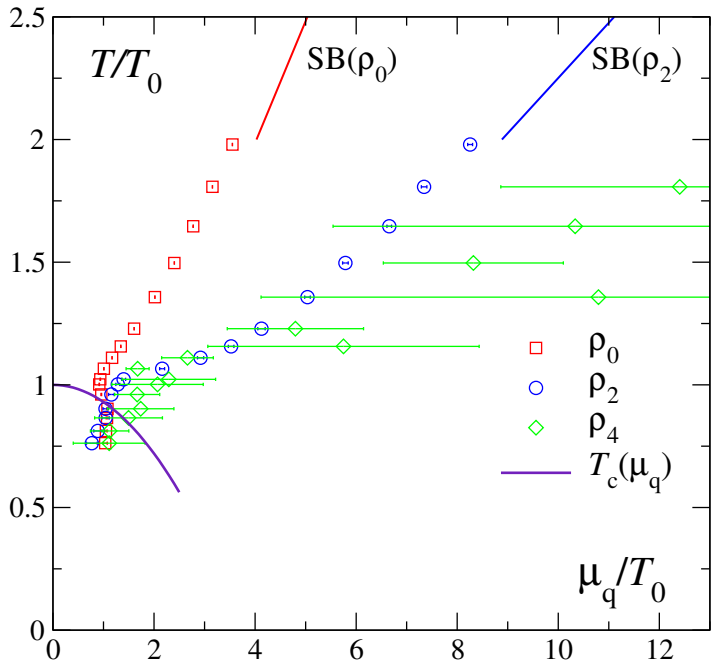

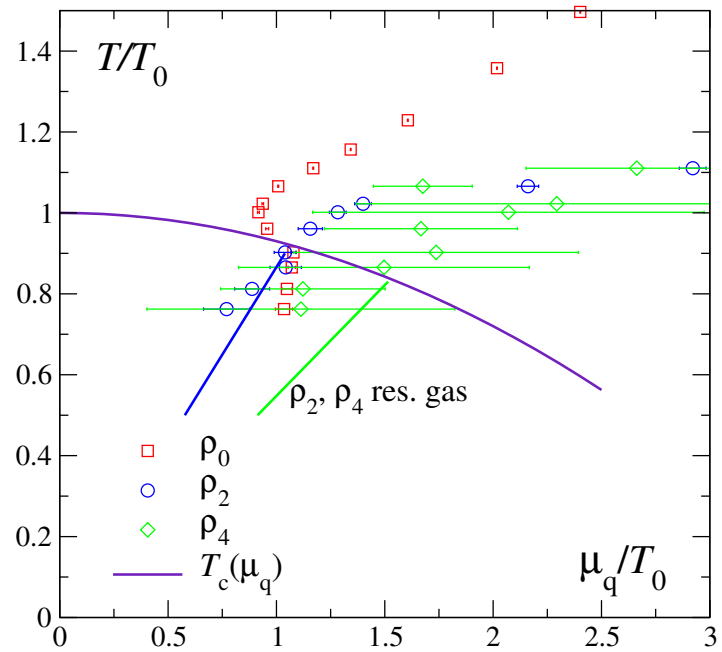

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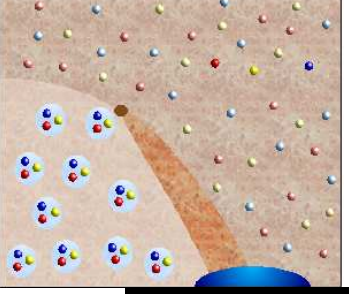
HOWEVER: see discussion of resonance gas!!!



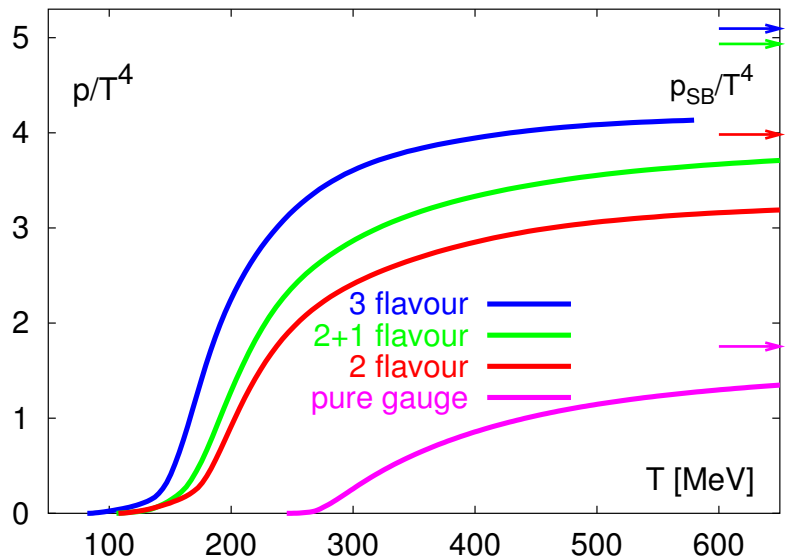
The pressure for $\mu_q/T > 0$

C.R. Allton et al. (Bielefeld-Swansea), PRD68 (2003) 014507)

$\mu_q = 0, \quad 16^3 \times 4$ lattice

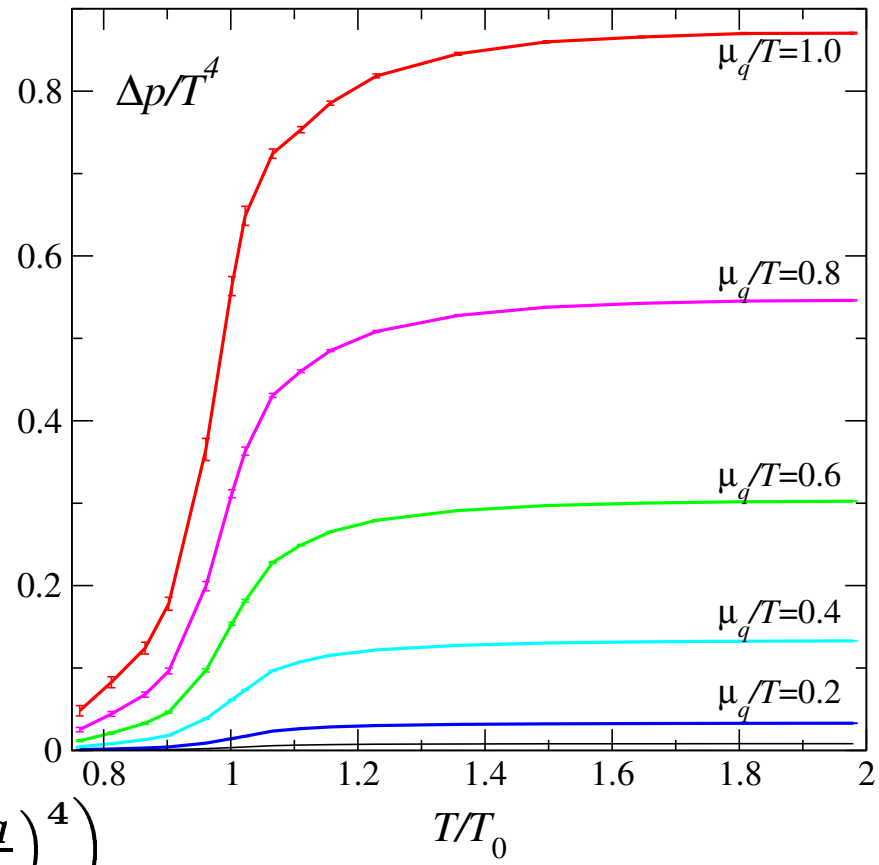
improved staggered fermions;

$n_f = 2,\ m_\pi \simeq 770\ MeV$



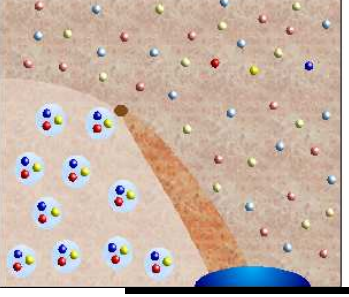
contribution from $\mu_q/T > 0$

Taylor expansion, $\mathcal{O}((\mu/T)^4)$



high-T, ideal gas limit

$$\frac{p}{T^4}\Big|_\infty = n_f\left(\frac{7\pi^2}{60} + \frac{1}{2}\left(\frac{\mu_q}{T}\right)^2 + \frac{1}{4\pi^2}\left(\frac{\mu_q}{T}\right)^4\right)$$



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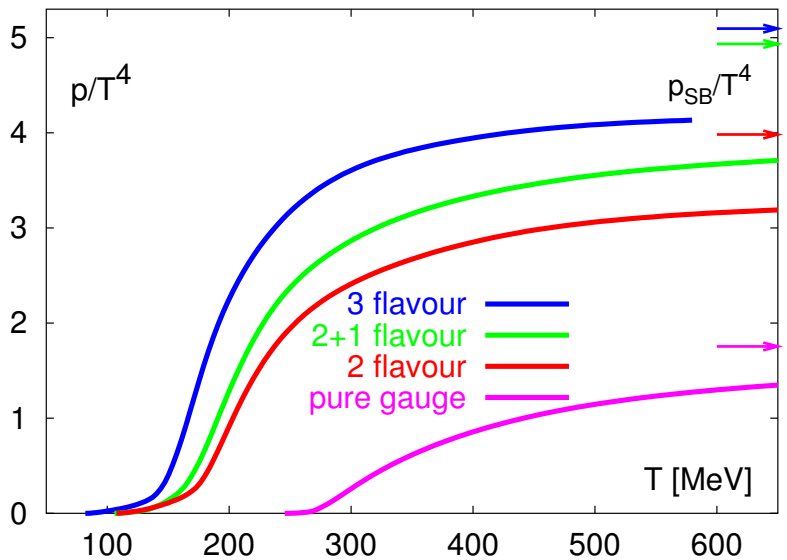
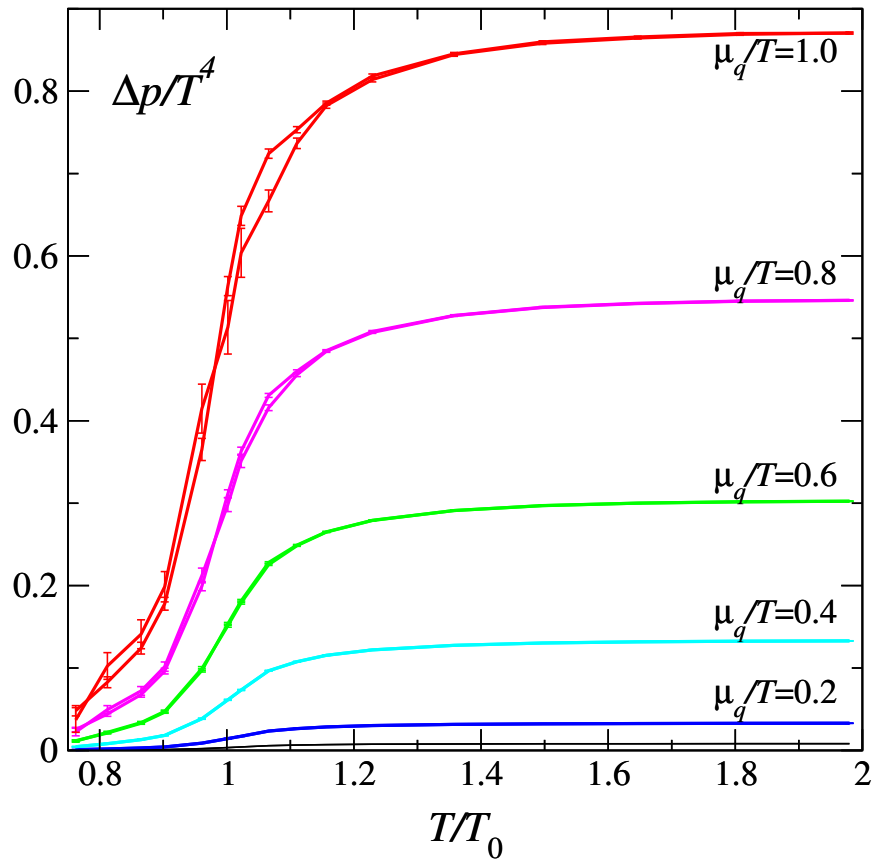


contribution from $\mu_q/T > 0$

NEW: Taylor expansion, $\mathcal{O}((\mu/T)^6)$

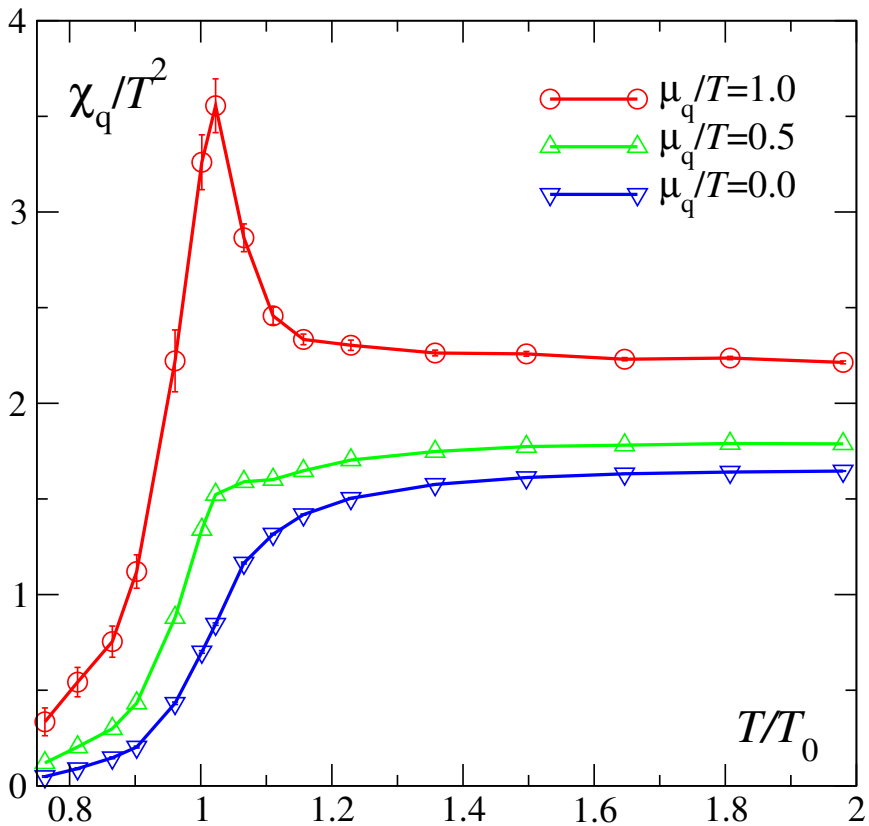


pattern for $\mu_q = 0$ and $\mu_q > 0$ similar;
quite large contribution in hadronic phase;
$\mathcal{O}((\mu/T)^6)$ correction small for $\mu_q/T \lesssim 1$

Fluctuations of the quark number density ($\mu_q > 0$)

quark number density fluctuations:

up to $\mathcal{O}\left((\mu_q/T)^2\right)$



$$\frac{\chi_q}{T^2} = \left(\frac{\partial^2}{\partial(\mu_q/T)^2}\frac{p}{T^4}\right)_{T\ \mathrm{fixed}}$$

high-T, massless limit: polynomial in (μ_q/T)

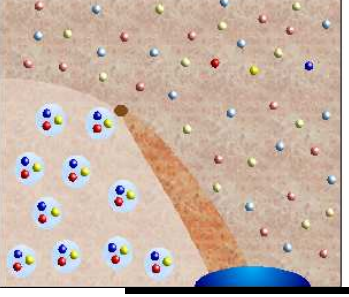
$$\frac{\chi_{q,SB}}{T^2} = n_f + \frac{3n_f}{\pi^2}\left(\frac{\mu_q}{T}\right)^2$$

large density fluctuations closer to the chiral critical point

$$\frac{\chi_q}{T^2} = \frac{1}{VT^3}\left(\langle N_q^2\rangle - \langle N_q\rangle^2\right)$$

$$\left(\frac{\partial p}{\partial n_q}\right)_T = \frac{n_q}{\chi_q}$$

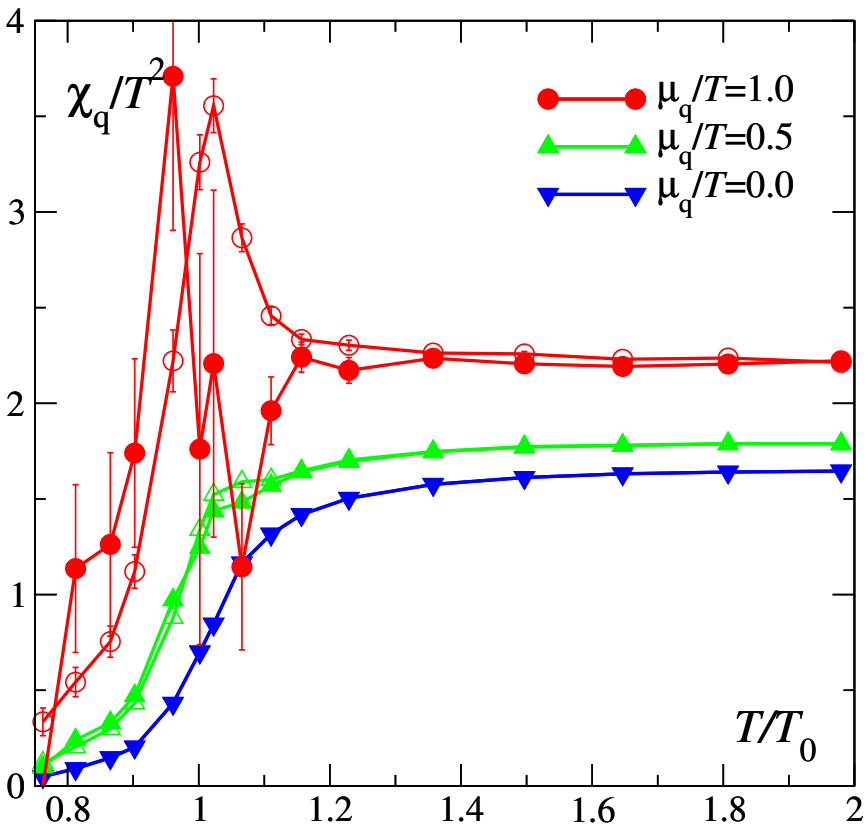
$\Rightarrow$ χ_q will diverge on chiral critical point



Fluctuations of the quark number density ($\mu_q > 0$)

quark number density fluctuations:

up to $\mathcal{O}\left((\mu_q/T)^4\right)$



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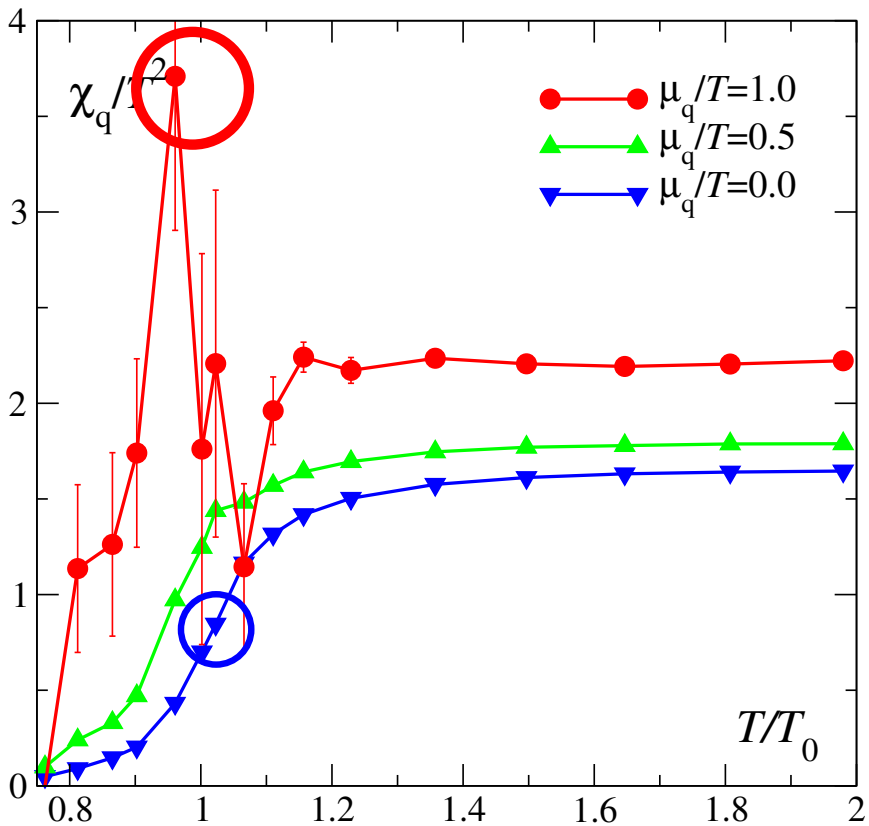
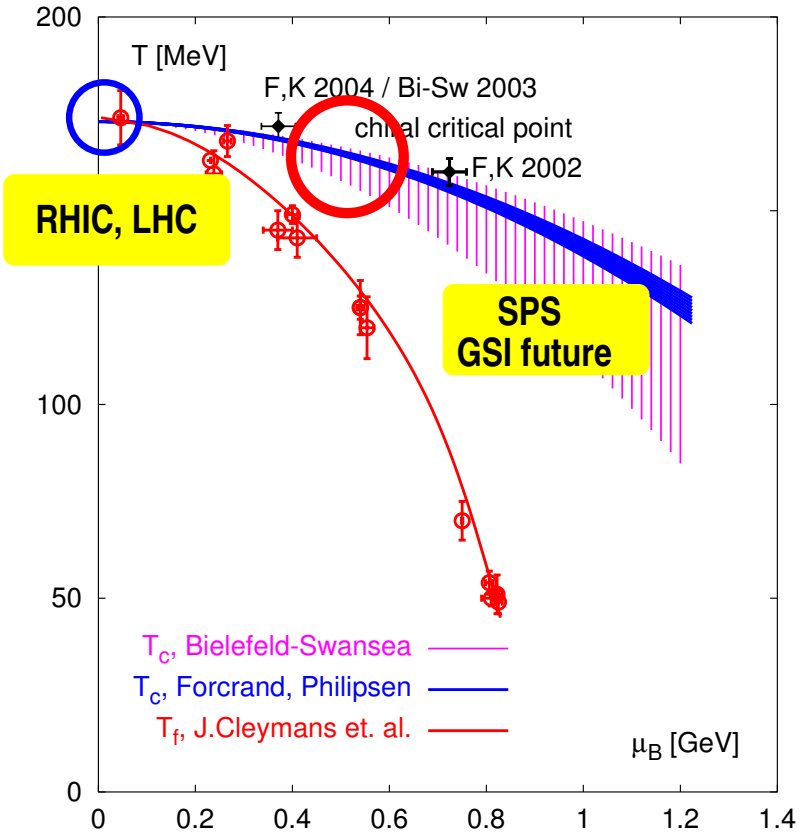
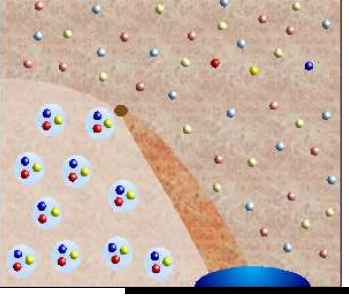


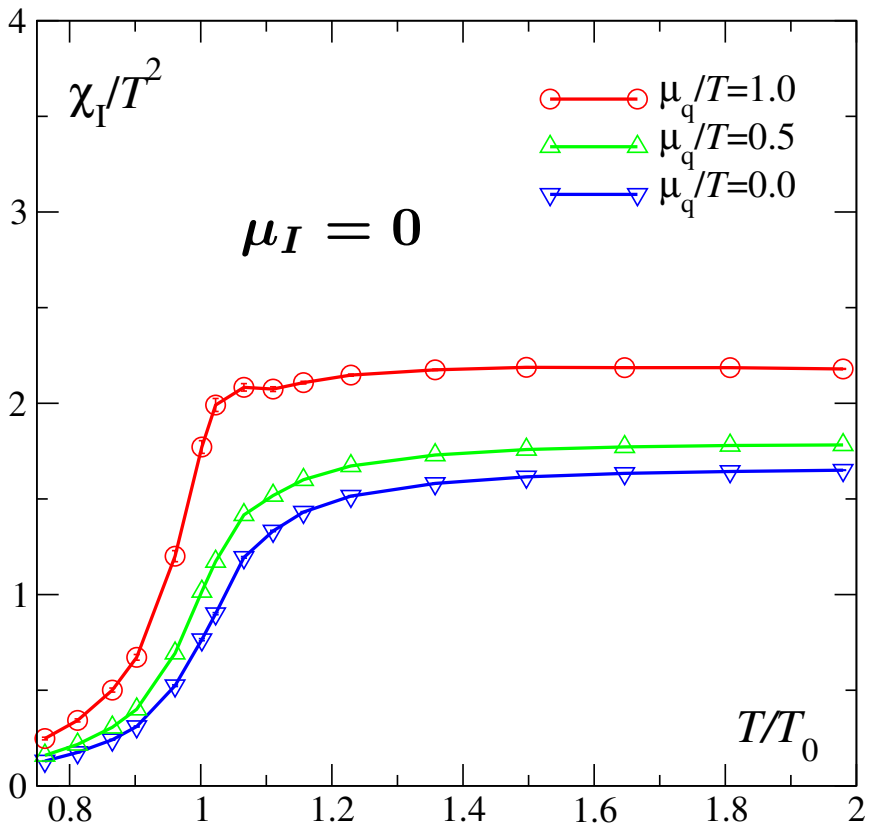




Fluctuations of the isospin and charge densities ($\mu_q > 0$, $\mu_I = 0$)

isospin density fluctuations:

up to $\mathcal{O}\left((\mu_q/T)^2\right)$



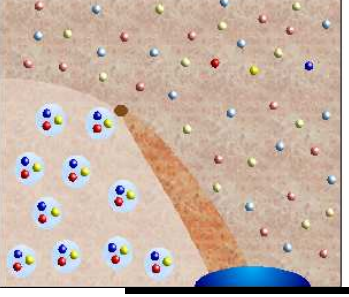
$$\frac{\chi_I}{T^2} = \left(\frac{\partial^2}{\partial(\mu_I/T)^2}\frac{p}{T^4}\right)_{T,\mu_q \text{ fixed}}$$

$\mu_I = 0$:

high-T, massless limit: polynomial in (μ_q/T)

$$\frac{\chi_{I,SB}}{T^2} \equiv \frac{\chi_{q,SB}}{T^2} = n_f + \frac{3n_f}{\pi^2}\left(\frac{\mu_q}{T}\right)^2$$

$$\frac{\chi_{I,SB}}{T^2} = \frac{1}{VT^3}\langle(N_u - N_d)^2\rangle$$



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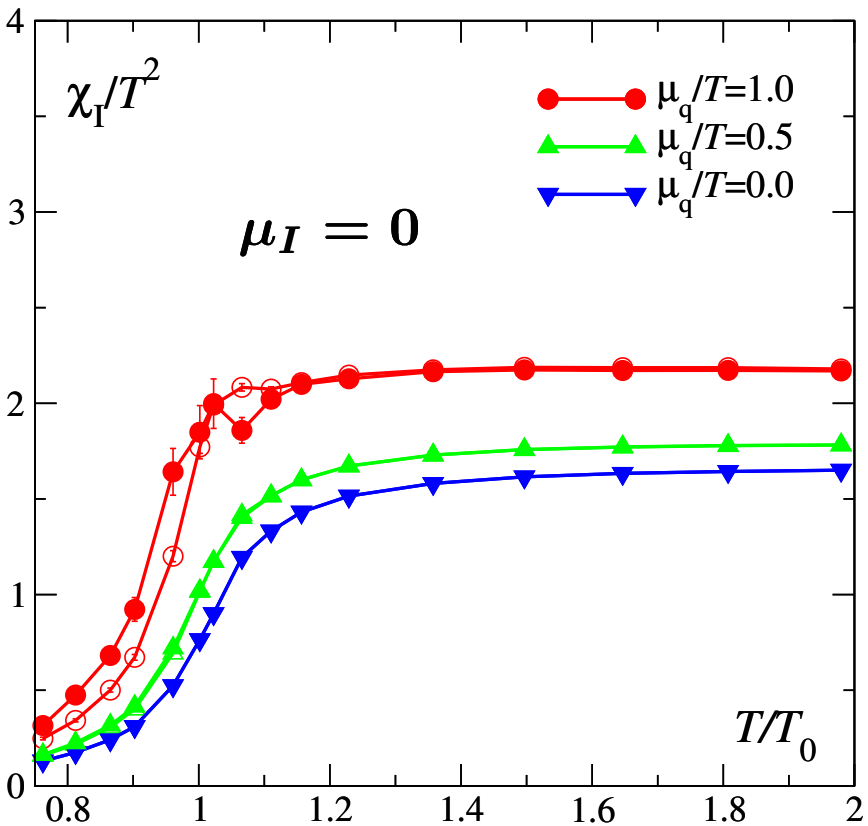

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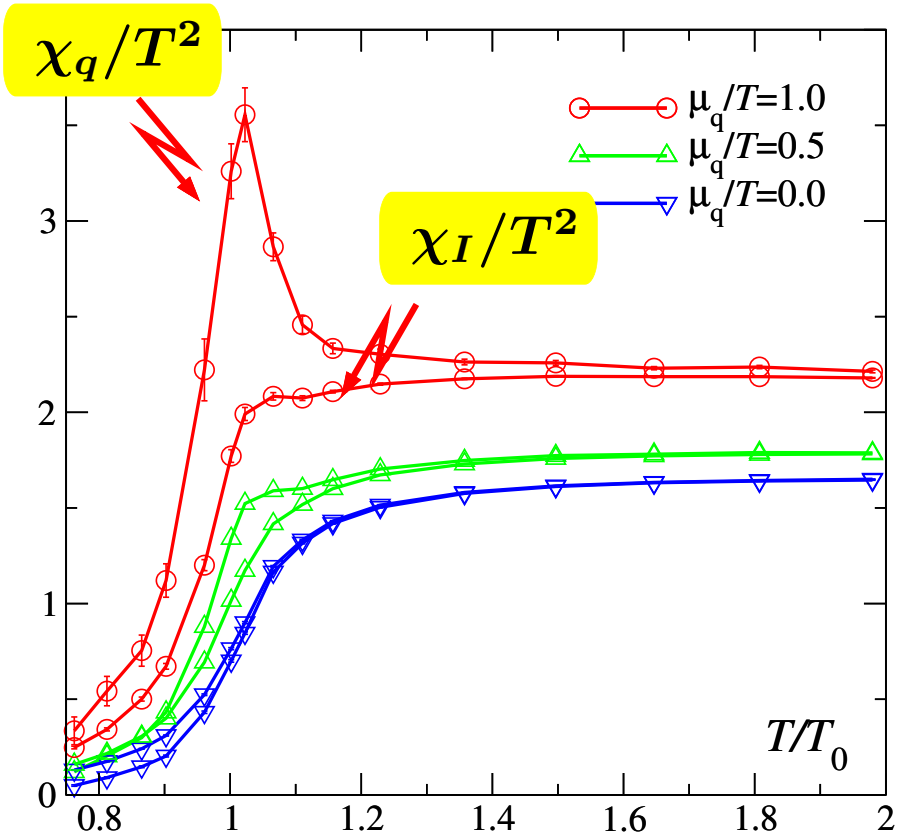
baryon number density and isospin density fluctuations

charge fluctuations



$$\chi_Q = \frac{1}{36}\chi_q + \frac{1}{4}\chi_I$$

$\sim \chi_q$ close to $T_c(\mu_c)$

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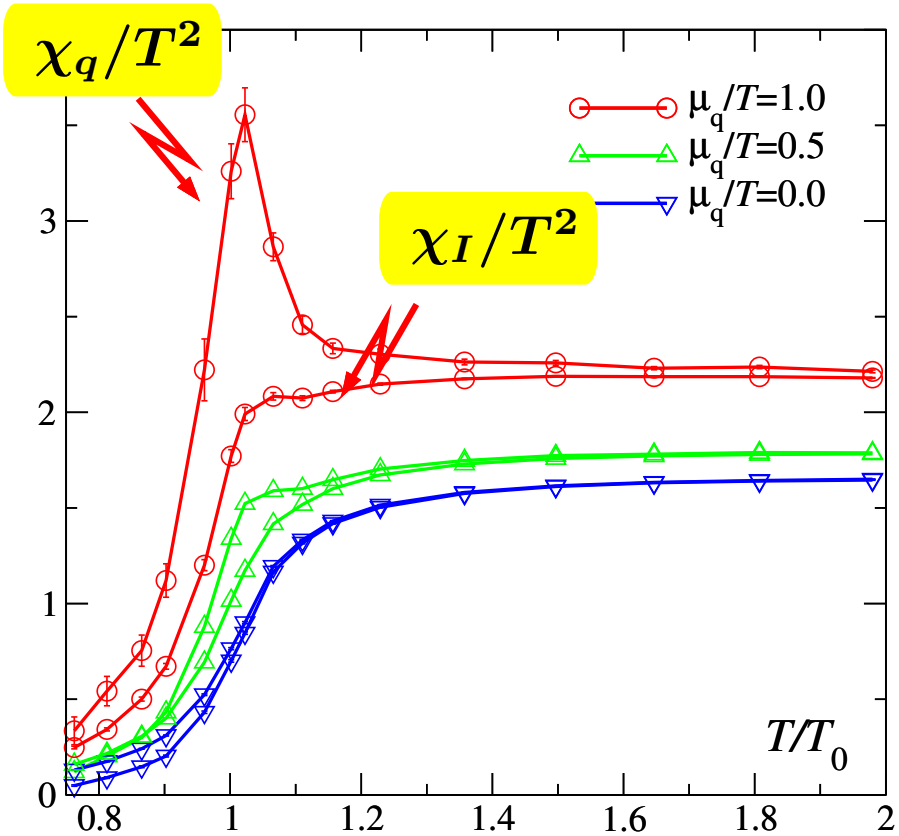


$$\chi_Q = \frac{1}{36}\chi_q + \frac{1}{4}\chi_I$$

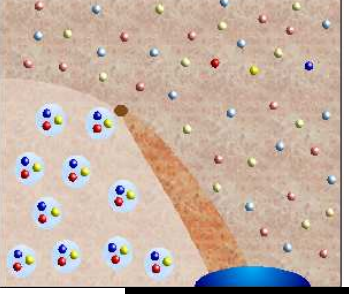
$\sim \chi_q$ close to $T_c(\mu_c)$

for comparison: proton number fluctuations;

$m_p/T_0 \simeq 5.5$, ideal gas

$\Rightarrow$ for $T \equiv T_0,\ \mu_q/T = 1$:

$$(\chi_q/T^2)^{proton} \simeq 1.2 \ll \chi_q/T^2$$



Critical behaviour in dense matter
Hagedorn's Resonance Gas

R. Hagedorn, Nuovo Cimento 35 (1965) 395

strong interactions $\Rightarrow$ exponentially rising spectrum of resonances

$$\textbf{Hagedorn spectrum}: \quad \rho(m_H) \sim c\, m_H^a\, \mathrm{e}^{m_H/T_H}$$

$$Z(T, \mu_B) = \int \mathrm{d}m_H\, \rho(m_H)\, \mathrm{e}^{-m_H/T}$$

$\Rightarrow$ critical behaviour: $T_c \equiv T_H$

(end of hadronic physics)

$$\rho(m_H) \equiv \rho_{\mathrm{meson}}(m_H, T) + \rho_{\mathrm{baryon}}(m_H, T, \mu_B)$$

- $\int \quad \Rightarrow \quad \sum$ experimentally known resonances
- $\rho(m_H) \quad \Rightarrow \quad \rho^{\mathrm{lat}}(m_H(m_\pi))$

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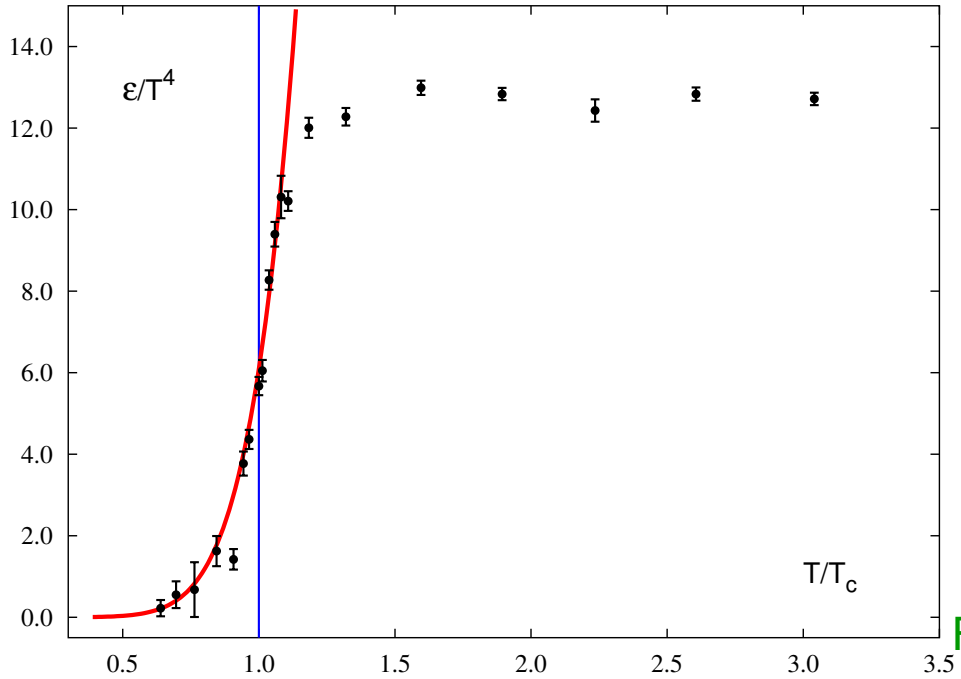


$\Rightarrow$ critical behaviour: $T_c \equiv T_H$

(end of hadronic physics)

resonance gas (adjusted mass spectrum):

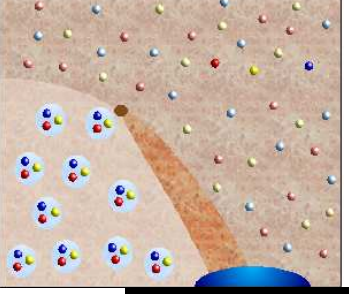
$\sim$ 1000 resonance d.o.f.

lattice calculation:

(2+1)-flavor QCD, $m_q/T = 0.4$, $\mu_B = 0$

continuous transition at T_c

FK, K. Redlich, A. Tawfik, hep-ph/0303108



Baryonic resonance gas $\Rightarrow$ Boltzmann approximation

heavy resonances, $T << m_H \quad \Rightarrow$ Boltzmann statistics

$$\mu_B \equiv 3\mu_q$$

thermodynamics: $p(T,\mu_B) = \frac{T}{V} \ln Z(T,\mu_B,V) = \sum_m p_m(T,\mu_B)$

$$\ln Z(T,\mu_B,V) = \sum_{i \in \text{mesons}} \ln Z^B_{m_i}(T,V) + \sum_{i \in \text{baryons}} \ln Z^F_{m_i}(T,\mu_B,V)$$

contribution of baryons with mass m

$$\frac{p_m}{T^4} = \frac{d}{\pi^2}\left(\frac{m}{T}\right)^2 \sum_{\ell=1}^{\infty} (-1)^{\ell+1}\, \ell^{-2}\, K_2(\ell m/T)\, \cosh(\ell\mu_B/T)$$

$$\Uparrow$$

$$K_2(x) \simeq \sqrt{\pi/2x}\, \exp(-x)\,, \quad x >> 1$$

- only $\ell = 1$ contributes for $(m_N - \mu_B) \gtrsim T$

$\Rightarrow$ Boltzmann approximation: $\frac{p_m}{T^4} = \frac{d}{\pi^2}\left(\frac{m}{T}\right)^2 K_2(m/T)\, \cosh(\mu_B/T)$

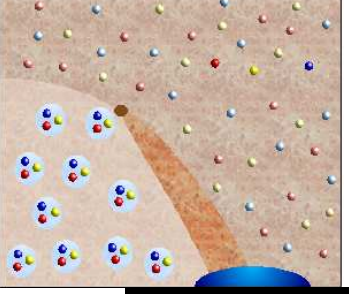
Boltzmann approximation with and without Taylor expansion

- Boltzmann approximation $\Rightarrow$ factorization of T and (μ/T)-dependence

		full result	Taylor expansion
pressure	$\frac{\Delta p}{T^4}$	$F(T)[\cosh\frac{\mu_B}{T}-1]$	$F(T)\left(\tilde{c}_2\left(\frac{\mu_q}{T}\right)^2+\tilde{c}_4\left(\frac{\mu_q}{T}\right)^4\right)$
number density	$\frac{n_q}{T^3}$	$3F(T)\sinh\frac{\mu_B}{T}$	$F(T)\left(2\tilde{c}_2\left(\frac{\mu_q}{T}\right)+4\tilde{c}_4\left(\frac{\mu_q}{T}\right)^3\right)$
quark number susceptibility	$\frac{\chi_q}{T^2}$	$9F(T)\cosh\frac{\mu_B}{T}$	$F(T)\left(2\tilde{c}_2+12\tilde{c}_4\left(\frac{\mu_q}{T}\right)^2\right)$

- $c_n = F(T)\,\tilde{c}_n$, with $\tilde{c}_2 = 9/2$, $\tilde{c}_4 = 27/8$ and $F(T) = \sum_i \frac{d_i}{\pi^2}\left(\frac{m_i}{T}\right)^2 K_2(m_i/T)$

- Taylor expansion only depends on $c_2(T)$ and spectrum independent ratios: c_n/c_2

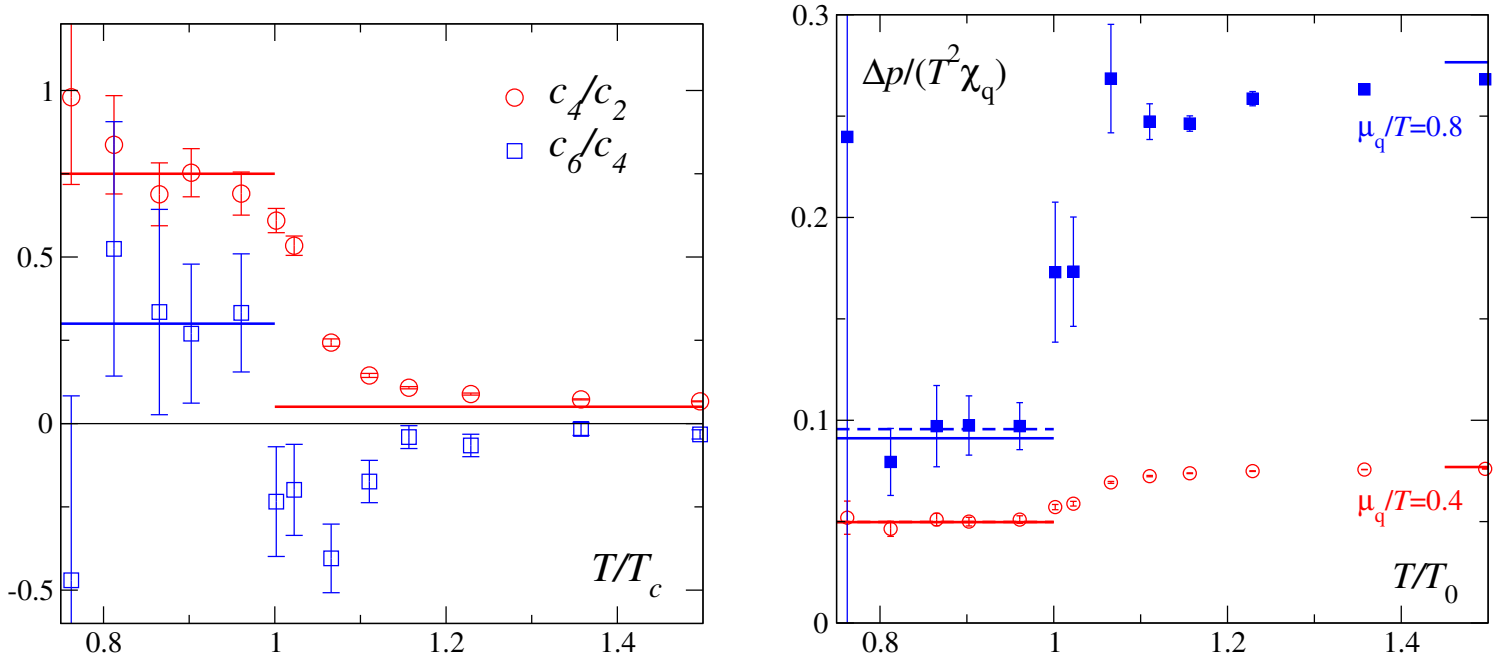


Resonance gas: spectrum independent consequences

FK, K. Redlich, A. Tawfik, PLB 571 (2003) 67

- Boltzmann approximation $\Rightarrow$ factorization $\Rightarrow$ temperature independent ratios; spectrum independent results

$$\frac{\Delta p}{T^2\chi_q} = \frac{1}{9}\left(1-\cosh^{-1}(3\mu_q/T)\right) \sim \frac{\left(\frac{\mu_q}{T}\right)^2 + \frac{c_4}{c_2}\left(\frac{\mu_q}{T}\right)^4 + \frac{c_6}{c_2}\left(\frac{\mu_q}{T}\right)^6}{2 + 12\frac{c_4}{c_2}\left(\frac{\mu_q}{T}\right)^2 + 30\frac{c_6}{c_2}\left(\frac{\mu_q}{T}\right)^4}$$



NEW: $\mathcal{O}(\mu^6)$

solid: exact; dashed $\mathcal{O}((\mu/T)^6)$

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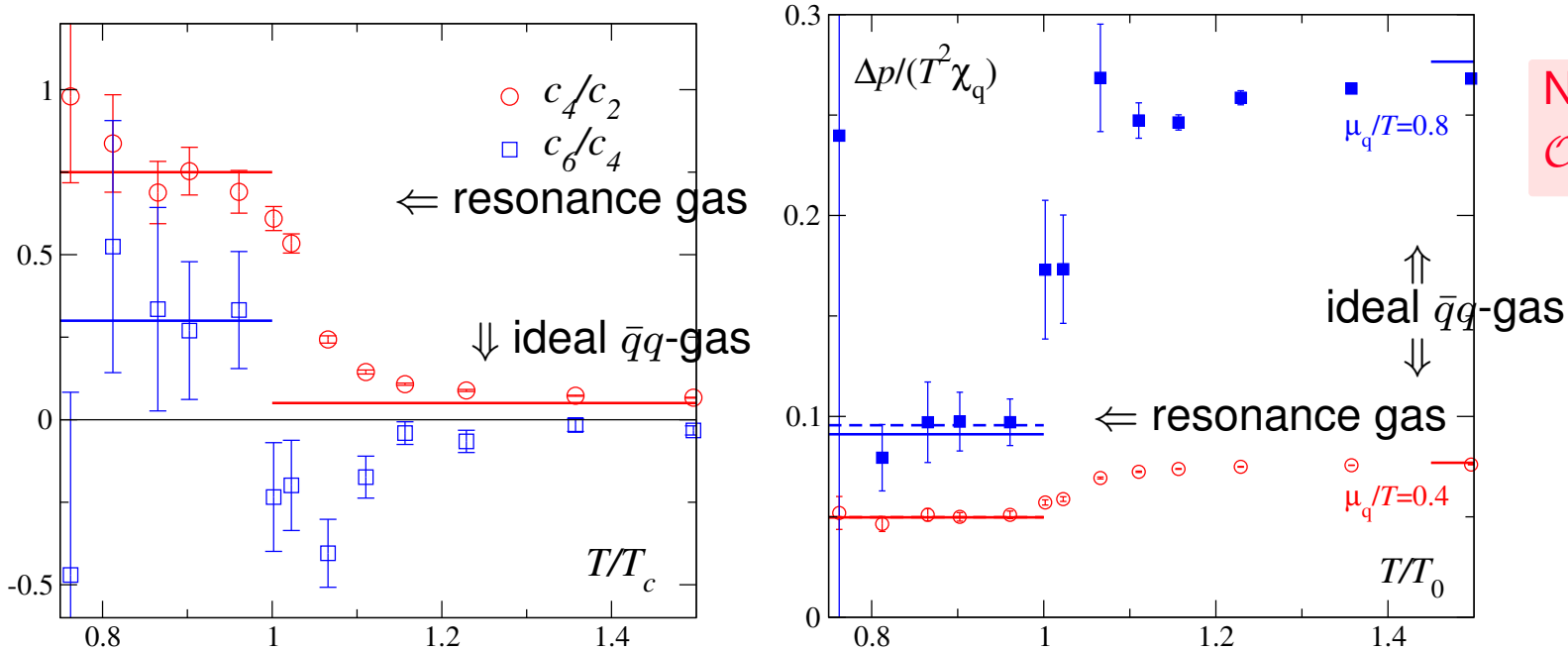


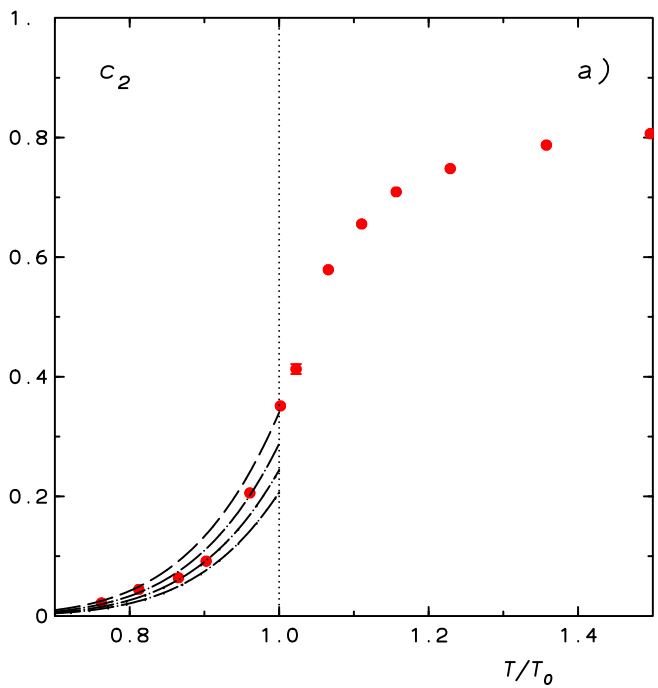
NEW: $\mathcal{O}(\mu^6)$

solid: exact; dashed $\mathcal{O}((\mu/T)^6)$

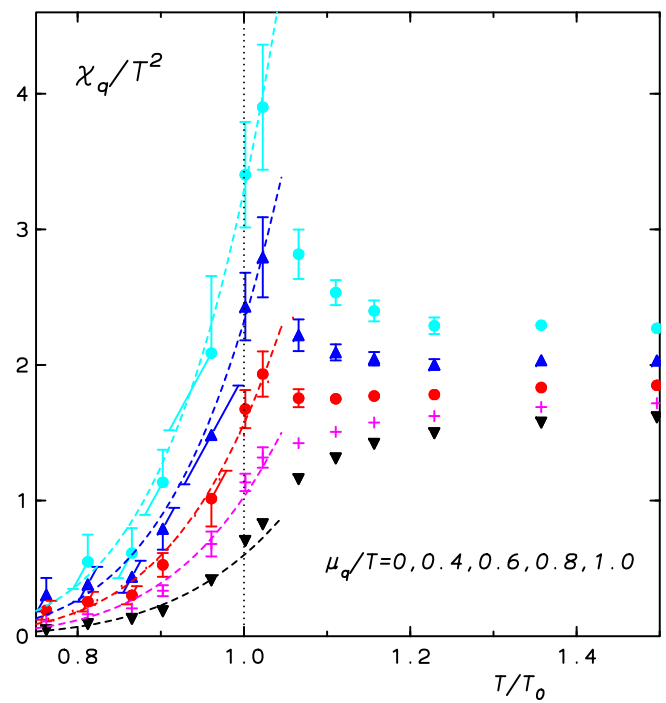
Resonance gas: spectrum dependent consequences

- "fit" with modified spectrum $m_H(m_\pi) = m_H(0) + A \left(\frac{m_\pi}{m_H(0)}\right)^2$
  $\Rightarrow$ tests factorization

$$\frac{\chi_q}{T^2} = 9F(T)\cosh(3\mu_q/T) \sim c_2(T)\left(2 + 12\,\frac{c_4}{c_2}\left(\frac{\mu_q}{T}\right)^2 + \mathcal{O}\left(\left(\frac{\mu_q}{T}\right)^4\right)\right)$$



$A = 0.9,\ 1.0,\ 1.1,\ 1.2$



$A = 1.0$

Critical behaviour and the resonance gas

- Resonance gas $\Rightarrow$ no critical behaviour, analytic, infinite radius of convergence
- finite density QCD $\Rightarrow$ expect 1st order phase transition for

$$T < T_c(\mu_c), \quad \frac{\mu_c}{T_c} \sim \mathcal{O}(1);$$

signaled by density fluctuations, i.e.

$\chi_q \to \infty$ for $\mu \to \mu_c$ at $T = T_c(\mu_c)$,

resonance gas approximation should break down close to the transition line;

1) $\dfrac{n_q}{T\chi_q} \to 0$ and $\dfrac{\Delta p}{T^2\chi_q} \to 0$ at the chiral critical point NO

2) $|c_{2n-2}/c_{2n}|$ should stay $\mathcal{O}(1)$ YES

SO FAR: no clear evidence for break-down of the resonance gas approximation below T_c

Strange Quark Matter 2004 ...

...topics that could not be discussed:

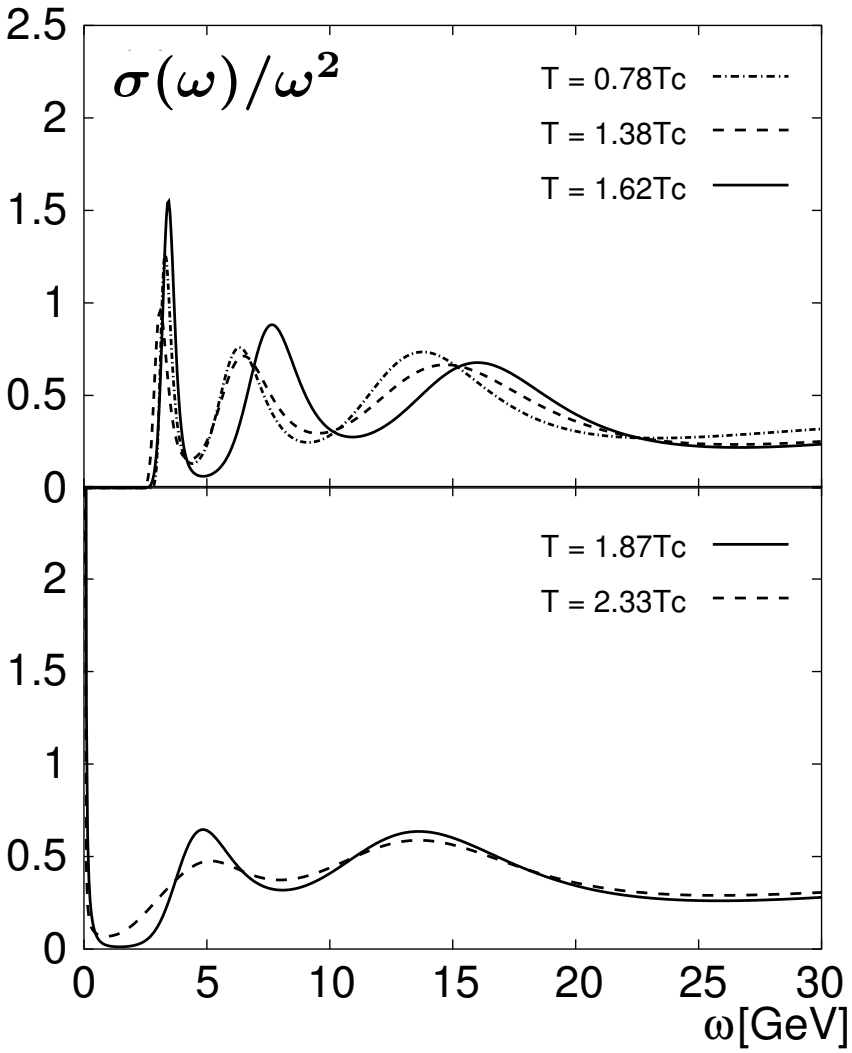
1) Heavy quark spectral functions
   - quarkonium suppression at high temperature
2) Heavy quark free energies and the heavy quark potential
   - renormalization of free energies;
   - running coupling at finite temperature

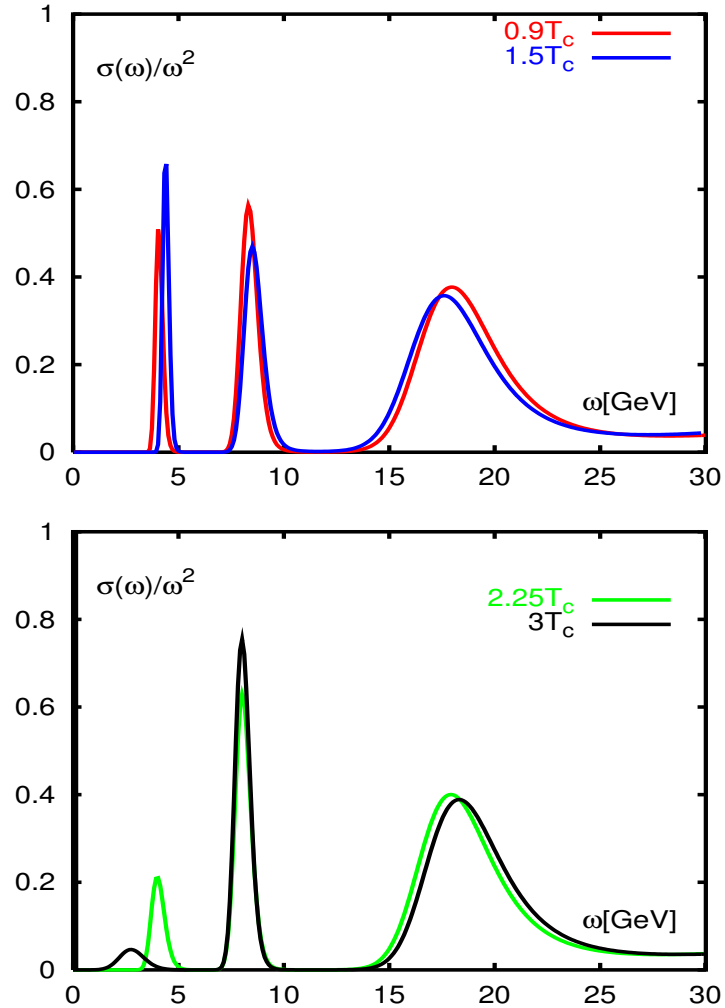
Heavy quark spectral functions
comparison of different approaches

M. Asakawa, T. Hatsuda, hep-lat/0308034

S. Datta et al., hep-lat/0312037

J/ψ spectral function





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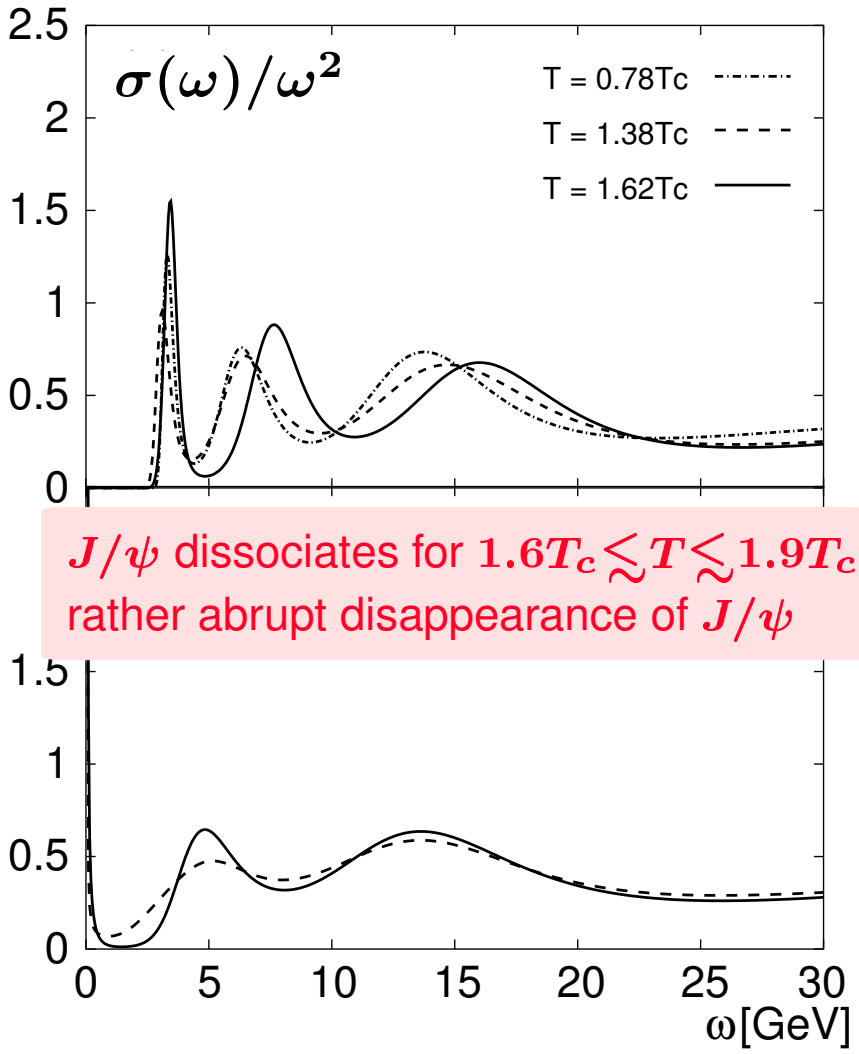

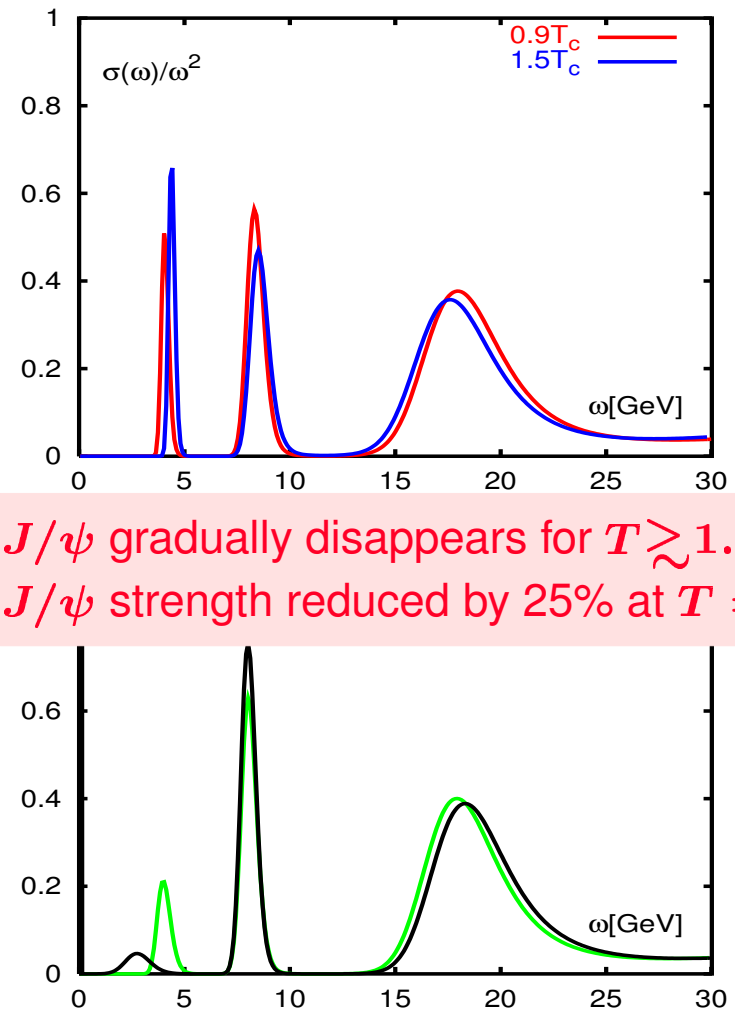

J/ψ dissociates for $1.6T_c \lesssim T \lesssim 1.9T_c$
rather abrupt disappearance of J/ψ

J/ψ gradually disappears for $T \gtrsim 1.5T_c$
J/ψ strength reduced by 25% at $T = 2.25T_c$

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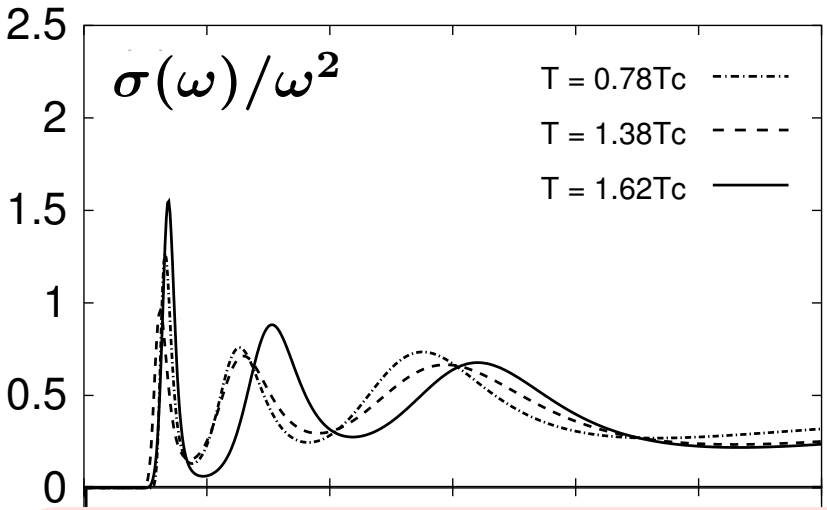

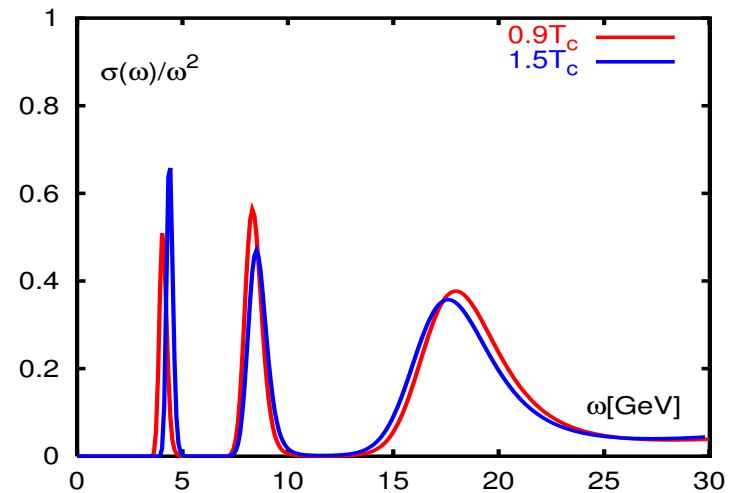

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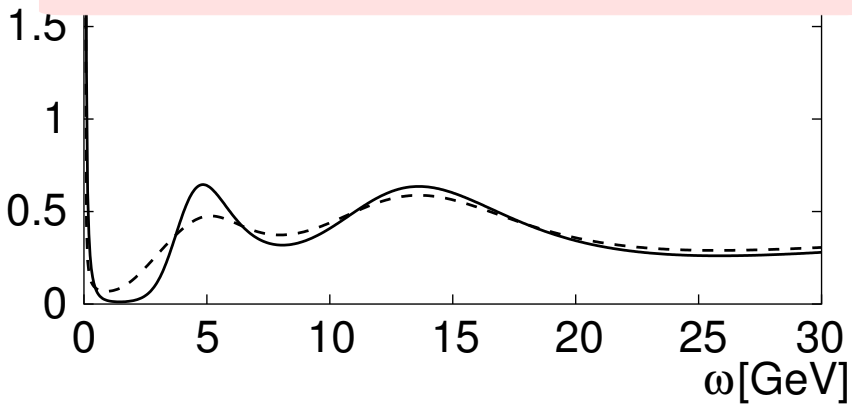
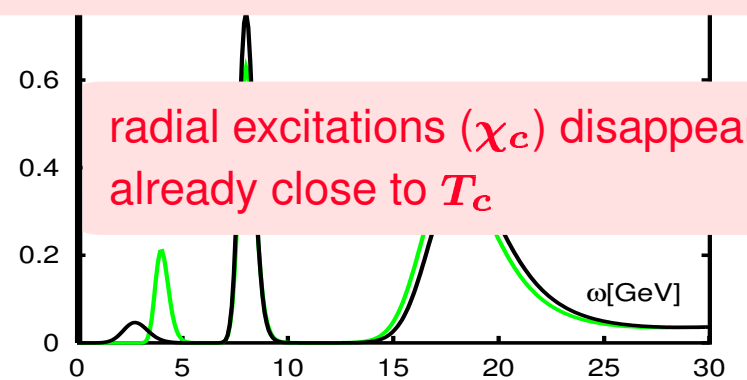
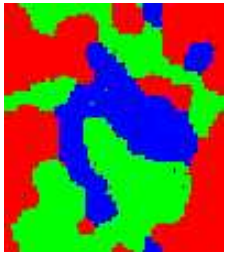




radial excitations (χ_c) disappear already close to T_c



Color averaged heavy quark free energies

O. Kaczmarek, FK, P. Petreczky, F.Zantow, PLB 543(2002)41

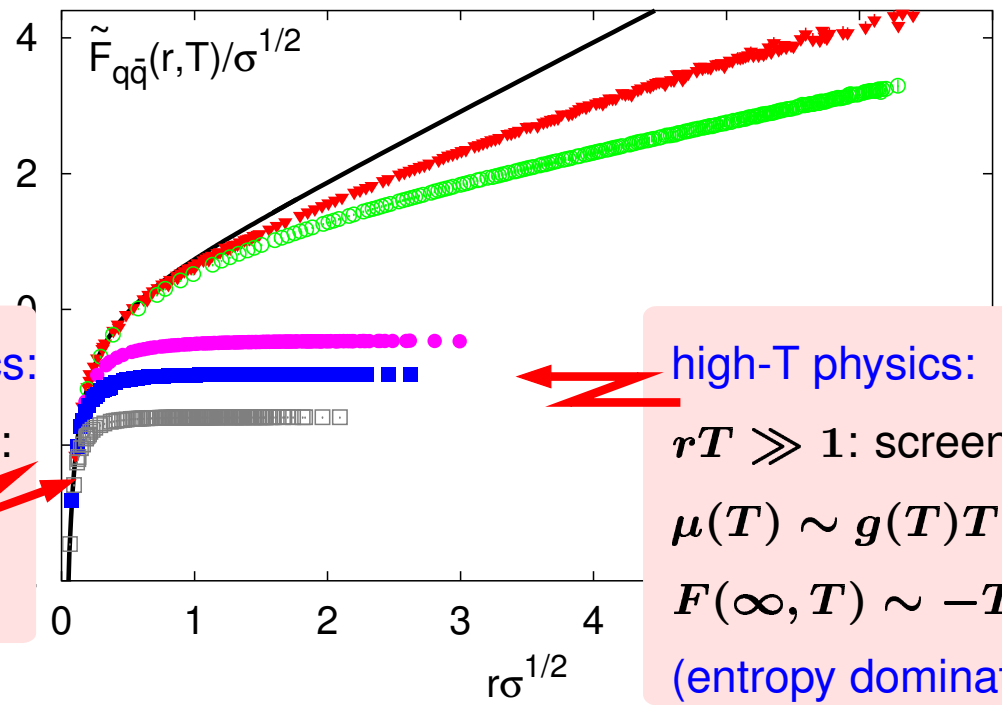
confined phase $(T < T_c)$: deconfined phase $(T > T_c)$:

matching ($\equiv$ renormalization) at short distances to $T = 0$ potential for all T

$$F = E - TS$$

$$E = -T^2 \frac{\partial F/T}{\partial T}$$

$$S = -\frac{\partial F}{\partial T}$$



$T = 0$, vacuum physics:

$rT \ll 1,\ r \ll 1/\sqrt{\sigma}$:

$F(r,T) \sim g^2(r)/r$

(energy dominated!!)

high-T physics:

$rT \gg 1$: screening

$\mu(T) \sim g(T)T$;

$F(\infty,T) \sim -T$

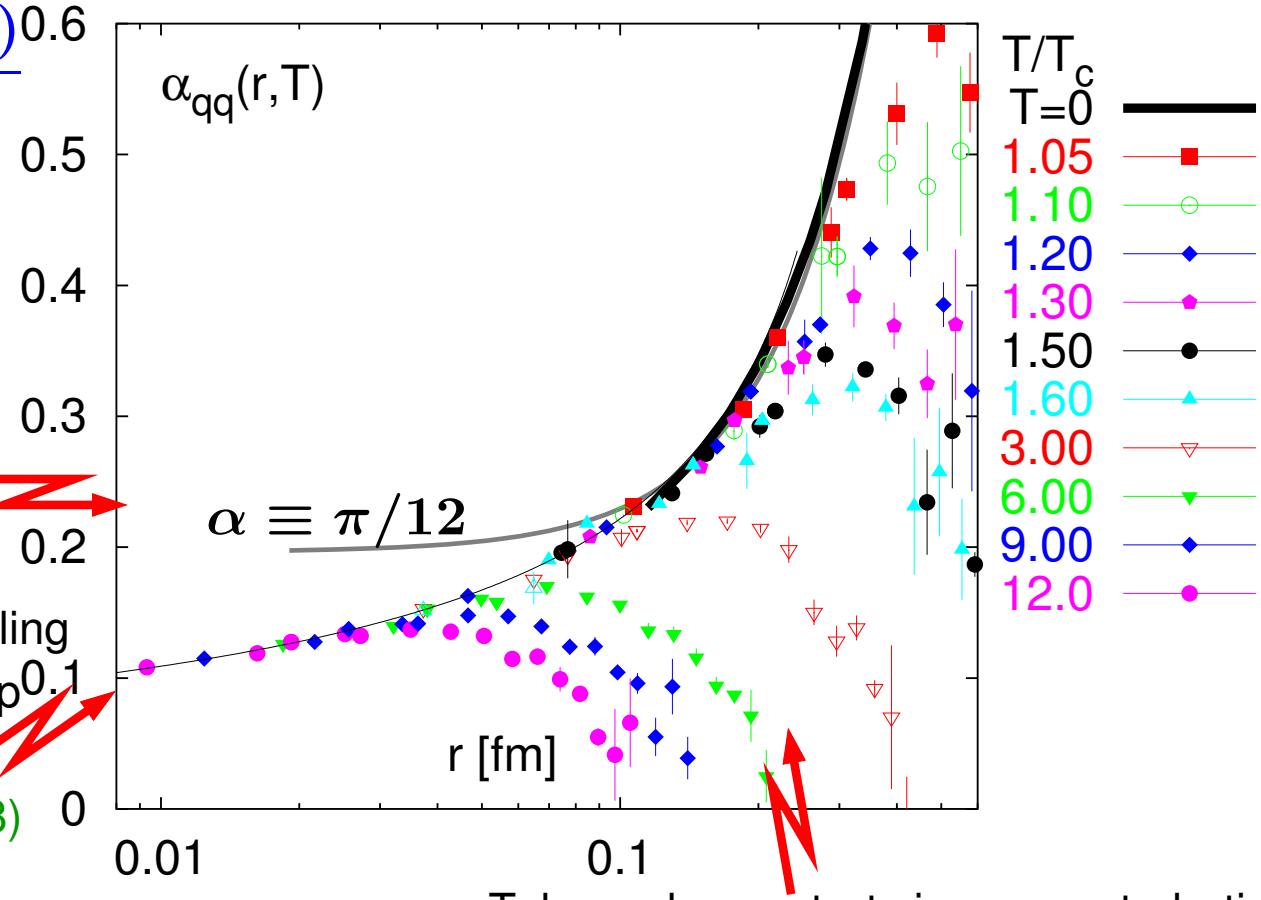
(entropy dominated!!)

Running coupling from singlet free energy

O. Kaczmarek, FK, P. Petreczky, F. Zantow, hep-lat/0406036

- singlet free energy defines a running coupling:

$$\alpha_{qq} = \frac{3r^2}{4} \frac{dF_1(r,T)}{dr}$$



large distance: constant Coulomb term (string model)

short distance: running coupling $\alpha(r)$ from $(T=0)$, 3-loop (S. Necco, R. Sommer, Nucl. Phys. B622 (2002) 328)

T-dependence starts in non-perturbative regime for $T \lesssim 3\ T_c$

- short distance physics $\Leftrightarrow$ vacuum physics

Conclusions

- QCD thermodynamics for small μ and large T can be analyzed reliably

  results on bulk thermodynamics in the hadronic phase so far agree quite well with resonance gas model calculations

- rapid rise of quark number fluctuations with increasing density (μ/T) is "normal" in the hadronic phase

  even larger fluctuations should signal presence of the chiral critical point;
  higher orders in the Taylor expansion needed;
  large density fluctuations call for large lattices

- location of the chiral critical point and curvature of $T_c(\mu)$ is uncertain to at least within a factor of 2.

  would like to see direct evidence for the first order transition

direct simulations at $\mu > 0$???