Effective restoration of chiral and axial symmetry at finite temperature and density

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Introduction / Formulation of the problem

Restoration of symmetries and deconfinement (QGP) are expected to occur under extreme conditions (ultra relativistic HIC, interior of neutron stars).

Questions:

1) Which symmetries are restored?

- $SU(N_f) \otimes SU(N_f)$ only?
- Both $SU(N_f) \otimes SU(N_f)$ and $U_A(1)$?
- Two possible scenarios in the last case **(Shuryak 94)**:
  (1) $T_C << T_{U(1)}$?
  (2) $T_C \simeq T_{U(1)}$?

2) Which observables should signal the restoration of symmetries?

Generalities about χ_S and $U_A(1)$ in QCD

- **QCD Lagrangian - symmetric under $U(N_f) \otimes U(N_f)$ transformations. Dynamical χ_S symmetry breaking:**

$$N_f = 3 \rightarrow \textbf{9 Goldstone bosons; } \eta' \textbf{ problem;}$$

$$+$$

- **Breaking of $U_A(1)$ at the quantum level (e.g by instantons):**

$$m'_\eta \simeq 1GeV; \textbf{ only 8 Goldstone bosons;}$$

$$+$$

- **Explicit breaking of $SU(3) \otimes SU(3)$:**

$$\Downarrow$$

Pseudoscalar and scalar mass spectrum;
Flavor mixing - violation of the OZI rule: θ_P, $\theta_S \neq$ from ideal mixing angles

Restoration of symmetries should have consequences on the spectrum of scalar and pseudoscalar mesons as well as in the mixing angles.

Effects of restoration of symmetries

Vanishing of symmetry violating observables

- χ_S : **quark condensates,** f_π
- $U_A(1)$: **topological susceptibility** χ

Note: when the current quark masses are different from zero the symmetry violating quantities go to zero asymptotically $\rightarrow$ only effective restoration of symmetries

Degenerescence of chiral partners

- **Whether** χ_S **and** $U_A(1)$ **are restored at the same point or not, when they get both restored,** a_0, σ, π^0, η, f_0, η',... **are all degenerated.**

Effective restoration of $U_A(1)$ symmetry with temperature

The topological susceptibility, χ, is essential to study the $U_A(1)$ problem: is related with $m_{\eta'}$ (Witten-Veneziano formula),

$$\frac{2N_f}{f_\pi^2}\chi_{pure}(0) = m_\eta^2 + m_{\eta'}^2 - 2m_K^2. \tag{1}$$

Several studies link the decrease of the topological susceptibility with restoration of $U_A(1)$ symmetry

Lattice results indicate a sharp drop of the topological susceptibility at the deconfining temperature; results with density are not yet available.

Model approaches:

- **Chiral models include an anomaly term that breaks the axial symmetry;**

- **Effective restoration of the axial symmetry may be achieved by assuming that the anomaly coefficient is a dropping function of the temperature:**
  - **phenomenological approach (Kunihiro 89)**
  - **lattice inspired (Schaffner-Bielich 00, Fukushima et. al. 01)**

Outline of the present work

1) We work in the $SU(3)$ NJL model, including the 't Hooft determinant;

2) We get the coefficient of the anomaly term as a function of T from χ, which is modeled as a Fermi function from lattice results (M. C. Chu et al., 95, B. Alles et al. 97)**;**

3) We extrapolate a similar approach to cold quark matter in weak equilibrium;

4) We discuss the restoration of χ_S and $U_A(1)$ by:

- **analyzing the convergence of scalar and pseudoscalar mesons (chiral partners);**
- **θ_S, θ_P mixing angles.**

Model and formalism

- $SU(3)$ **NJL model with 't Hooft determinant.**

The Lagragian is:

$$\begin{aligned}\mathcal{L} &= \bar{q}\,(i\partial\cdot\gamma - \hat{m})\,q + \frac{g_S}{2}\sum_{a=0}^{8}[(\bar{q}\lambda^a q)^2 + (\bar{q}(i\gamma_5)\lambda^a q)^2] \\ &+ g_D[\det[\bar{q}(1-\gamma_5)q] + \det[\bar{q}(1-\gamma_5)q]]\,. \end{aligned} \tag{2}$$

Gap Equations:

$$\boxed{M_i = m_i - 2\,g_S <<\bar{q}_i\,q_i>> - 2\,g_D <<\bar{q}_j\,q_j>><<\bar{q}_k\,q_k>>} \tag{3}$$

Meson effective action $\Longrightarrow$ **Meson Propagators,** $g_{M\bar{q}q}$, $f_{M\bar{q}q}$,...

Two cases will be studied:

- **Finite temperature and zero density;**
- **Asymmetric quark matter with strange quarks in** β **equilibrium, at zero** T**.**

- **P. Costa et al., EPL (02), P. Costa et al., PLB (03), P. Costa et al., PRC (04)**

π^0 *and* a_0 *propagators*

The masses of the π^0 and a_0 mesons can be determined via the condition

$$\boxed{\left(1 - K_{P,S}\ J_{uu}(P_0 = m_{\pi^0,a_0}, \mathbf{P} = 0)\right) = 0} \tag{4}$$

with:

- $$K_P = g_S + g_D << \bar{s}s >>; \tag{5}$$
- $$K_S = g_S - g_D << \bar{s}s >>; \tag{6}$$
- $$J_{ii}(P_0) = 4(2I_1^i(T,\mu_i) + P_0^2 I_2^{ii}(P_0,T,\mu_i)); \tag{7}$$

η *and* σ *propagators:*

The masses of the η and σ mesons can be determined via the condition

$$D_{\eta,\sigma}^{-1}(m_{\eta,\sigma}, \mathbf{0}) = 0\,, \tag{8}$$

where

$$\boxed{D_{\eta,\sigma}^{-1} = (\mathcal{A} + \mathcal{C}) - \sqrt{(\mathcal{C} - \mathcal{A})^2 + 4\mathcal{B}^2}.} \tag{9}$$

η' *and* f_0 *propagators*

The masses of the η' and f_0 mesons can be determined via the condition

$$D^{-1}_{\eta,f_0}(m_{\eta,f_0},\mathbf{0}) = 0\,, \tag{10}$$

where

$$\boxed{D^{-1}_{\eta,f_0} = (\mathcal{A}+\mathcal{C}) - \sqrt{(\mathcal{C}+\mathcal{A})^2 + 4\mathcal{B}^2},} \tag{11}$$

with

- $$\begin{cases} \mathcal{A} = P_{88} - \Delta\Pi_{00}; \\ \mathcal{B} = -P_{08} - \Delta\Pi_{08}; \\ \mathcal{C} = P_{00} - \Delta\Pi_{88}; \\ \Delta = P_{00}P_{88} - P_{08}^2; \end{cases}$$

- $$\begin{cases} P_{00} = g_S \mp \frac{2}{3} g_D \left(<< \bar{q}_u\, q_u >> + << \bar{q}_d\, q_d >> + << \bar{q}_s\, q_s >> \right); \\ P_{08} = \pm \frac{1}{3\sqrt{2}} g_D \left(<< \bar{q}_u\, q_u >> + << \bar{q}_d\, q_d >> -2 << \bar{q}_s\, q_s >> \right); \\ P_{88} = g_S \pm \frac{1}{3} g_D \left(2 << \bar{q}_u\, q_u >> +2 << \bar{q}_d\, q_d >> - << \bar{q}_s\, q_s >> \right); \end{cases}$$

- $$\begin{cases} \Pi_{00} = \frac{2}{3} \left(I_{uu}(m^2) + I_{dd}(m^2) + I_{ss}(m^2) \right); \\ \Pi_{08} = \frac{\sqrt{2}}{3} \left(I_{uu}(m^2) + I_{dd}(m^2) - 2I_{ss}(m^2) \right); \\ \Pi_{88} = \frac{1}{3} \left(I_{uu}(m^2) + I_{dd}(m^2) + 4I_{ss}(m^2) \right); \end{cases}$$

The topological susceptibility

The topological susceptibility is given by

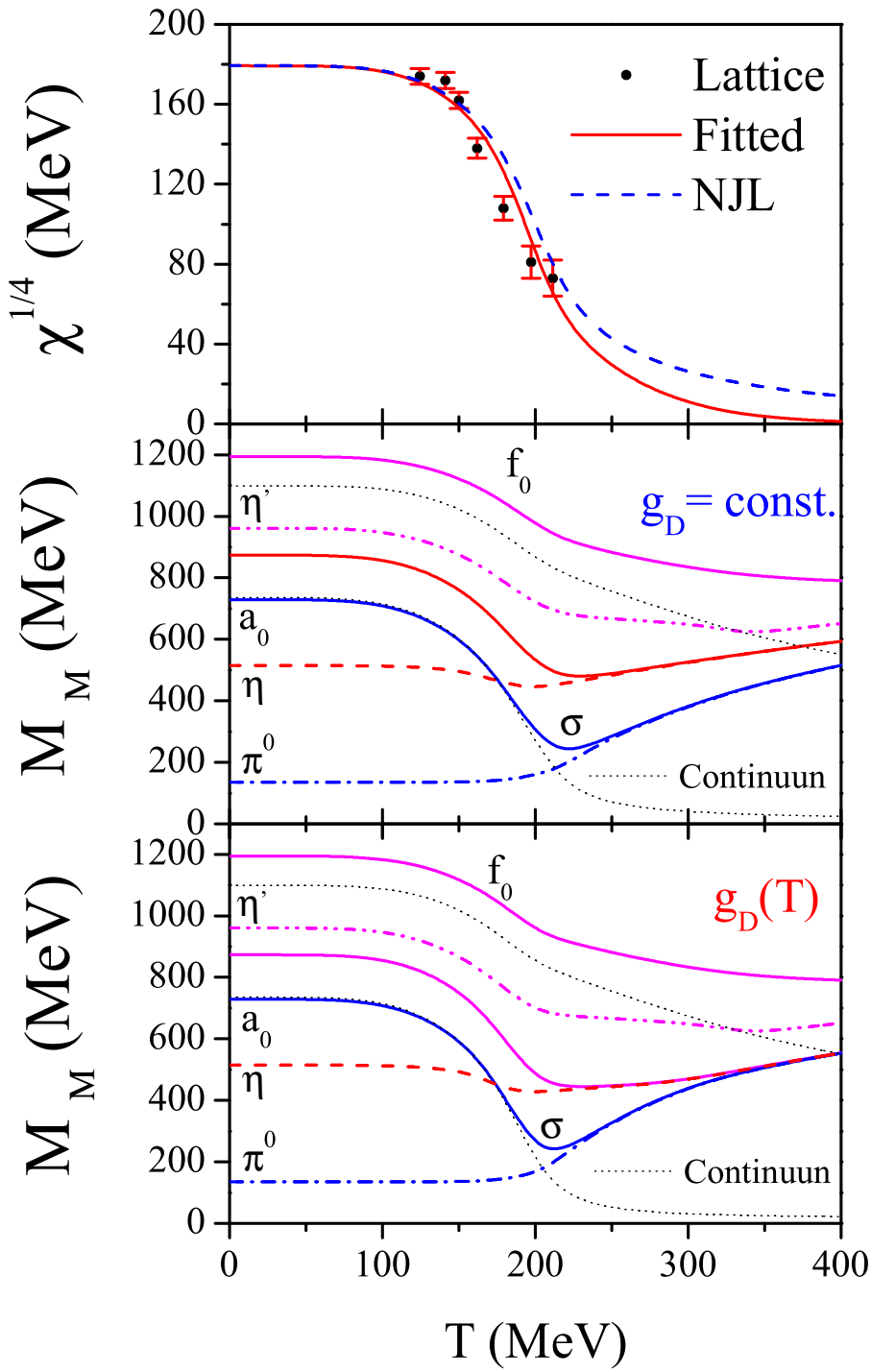
$$\chi(k^2) = \int \mathrm{d}^4x \; \mathrm{e}^{-ikx} \langle 0|TQ(x)Q(0)|0\rangle_{\mathrm{connected}}, \tag{12}$$

where $Q(x)$ is the topological charge density.

The general expression for χ in NJL model has been obtained by Fukushima *et al.* 01:

$$\begin{aligned}
\chi \;=\; & 4g_D^2\Big[2\Pi_{uu}(0) < \bar{q}_u\, q_u >^2 < \bar{q}_s\, q_s >^2 + \Pi_{ss}(0) < \bar{q}_u\, q_u >^4 \\
& + \Big\{\frac{1}{\sqrt{3}} < \bar{q}_u\, q_u > (< \bar{q}_s\, q_s > - < \bar{q}_u\, q_u >) \begin{pmatrix} \Pi_{88} \\ \Pi_{80} \end{pmatrix}^t \\
& + \frac{1}{\sqrt{6}} < \bar{q}_u\, q_u > (2 < \bar{q}_s\, q_s > + < \bar{q}_u\, q_u >) \begin{pmatrix} \Pi_{08} \\ \Pi_{00} \end{pmatrix}^t \Big\} \\
& \times\; 2\hat{K}\left(1 - 2\hat{\Pi}\hat{K}\right)^{-1} \\
& \times\; \Big\{\frac{1}{\sqrt{3}} < \bar{q}_u\, q_u > (< \bar{q}_s\, q_s > - < \bar{q}_u\, q_u >) \begin{pmatrix} \Pi_{88} \\ \Pi_{08} \end{pmatrix} \\
& + \frac{1}{\sqrt{6}} < \bar{q}_u\, q_u > (2 < \bar{q}_s\, q_s > + < \bar{q}_u\, q_u >) \begin{pmatrix} \Pi_{80} \\ \Pi_{00} \end{pmatrix} \Big\}\Big].
\end{aligned} \tag{13}$$

Results at $T \neq 0$ *and* $\rho = 0$



- $T \approx 250$ **MeV: degenerescence between the chiral partners** (π, σ) **and** (a_0, η)

$$\Downarrow$$

Effective restoration of $SU(2)$ **symmetry**

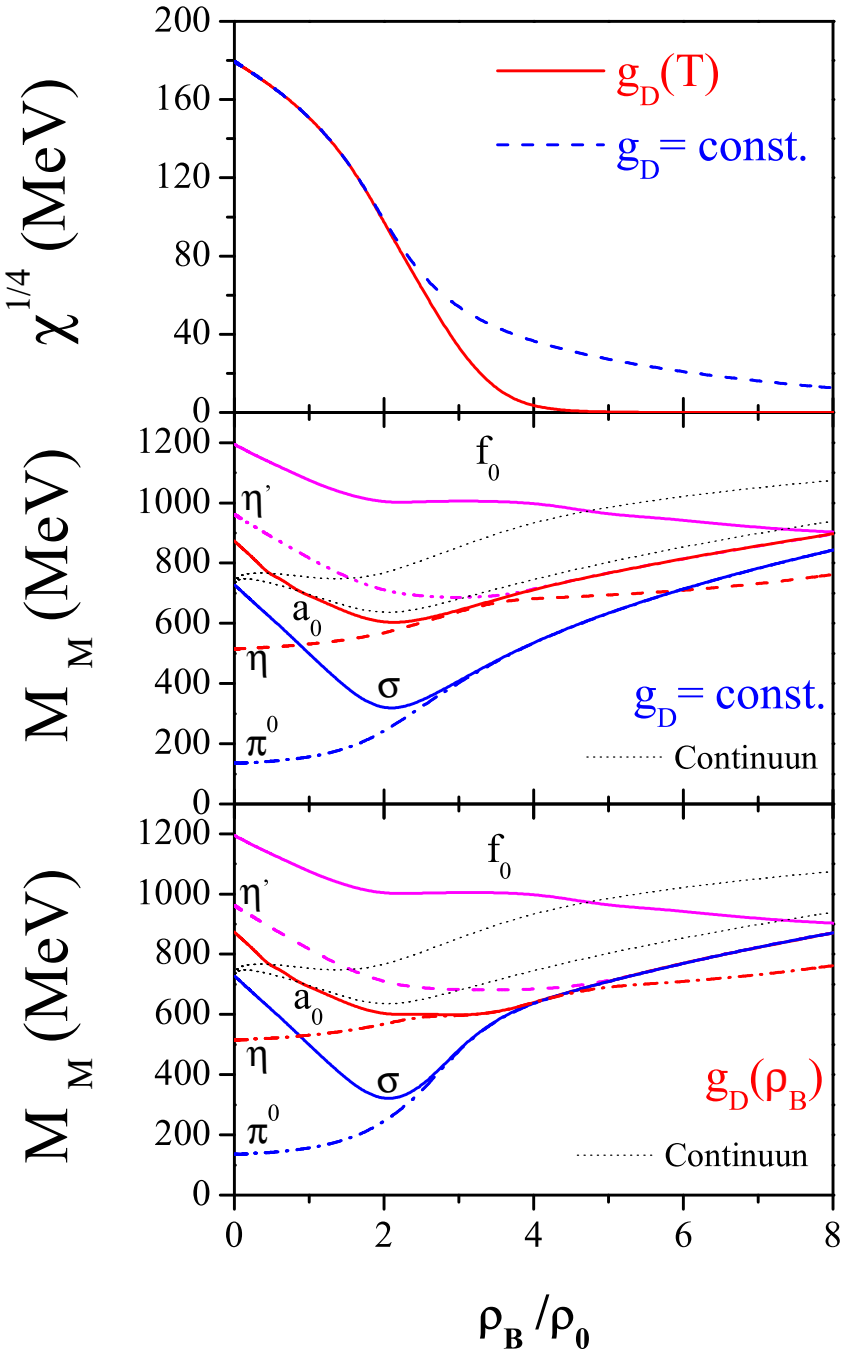
- $T \approx 350$ **MeV:**
  - $(a_0,\ \sigma)$ **become degenerate with** $(\pi,\ \eta)$**, showing an effective restoration of both** $SU(2)$ **and axial symmetries.**
  - $\chi \rightarrow 0$

$$\Downarrow$$

Effective restoration of both chiral and axial symmetries

- **Without the restoration of** $\mathrm{U}_A(1)$ **symmetry:**
  - a_0 **mass never meets the** π^0 **mass**
  - σ **mass never meets the** η **mass**
- η' **and** f_0 **masses do not show a clear tendency to converge**

Results at $T = 0$ and $\rho \neq 0$



- $\rho_B \approx 3.0\rho_0$: **degenerescence between the chiral partners** (π, σ) **and** (a_0, η)

$$\Downarrow$$

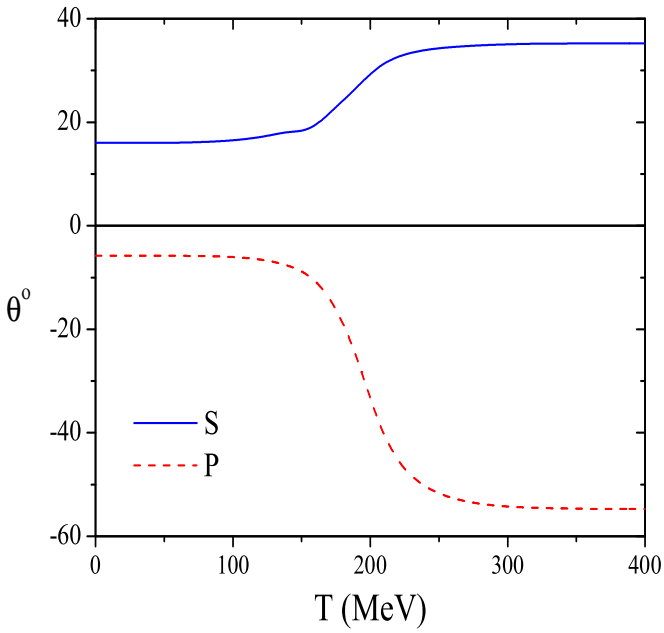
Effective restoration of $SU(2)$ **symmetry**

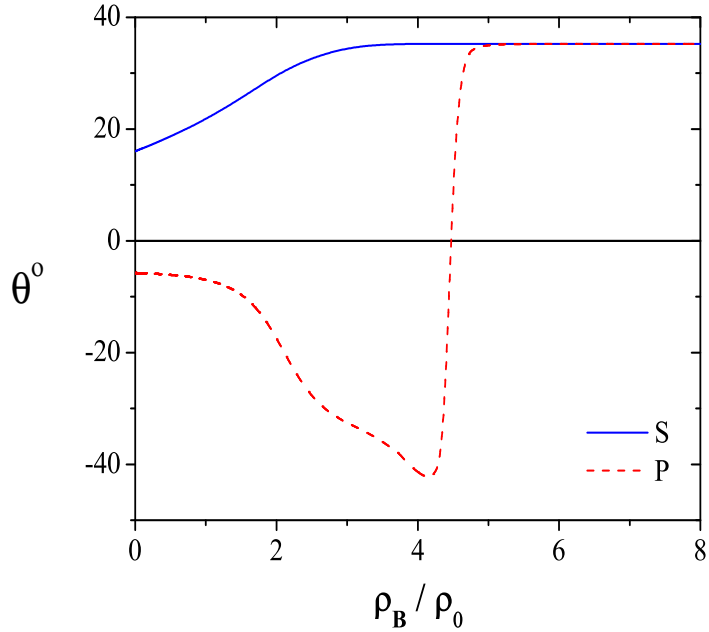
- $\rho_B \geq 4.0\rho_0$**:**
  - $\chi \to 0$
  - $4.0\rho_0 \leq \rho_B \leq 4.8\rho_0$**:**
    - $*$ $(a_0,\ \sigma)$ **become degenerate with** $(\pi,\ \eta)$
  - $\rho_B > 4.8\rho_0$**:**
    - $*$ η **and** η' **change identities**
    - $*$ $(a_0,\ \sigma)$ **become degenerate with** $(\pi^0,\ \eta')$

$$\Downarrow$$

Effective restoration of both chiral and axial symmetries

Mixing angles:





- **In both cases mixing angles $\Rightarrow$ ideal values, at ρ, T where $U_A(1)$ symmetry is effectively restored**

- **Change of identities of η and η' at $\rho_B > 4.8\rho_0$: θ_P changes sign**

Summary

- **We have explored a framework to investigate possible restoration of chiral and axial symmetries with temperature and with density;**

- **We implement a criterion which combines a lattice-inspired behavior of the topological susceptibility with the convergence of chiral partners;**

- $SU(2) \otimes SU(2)$ **is restored at moderate temperature (density);**

- $U_A(1)$ **is restored when** σ, π^0, a_0 **and** $\eta(\eta')$ **are degenerated, and** $\chi \to 0$;

- **Our results are compatible with Shuryak's scenario (1)** ($T_C << T_{U(1)}$);

- **Full restoration of** $U(3) \otimes U(3)$ **are not yet visible.**