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$\pi^0, \eta \to \gamma\gamma$ *at finite temperature and density*

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- **Motivation;**
- **Model and formalism;**
- π^0 **propagator;**
- η **propagator;**
- **The decay** $M \rightarrow \gamma\gamma$**;**
- **Summary.**

Motivation:

What can we learn from the decays $\pi^0(\eta) \rightarrow \gamma\gamma$ **at** $T \neq 0$ / $\rho \neq 0$ **?**

1) Pseudoscalar mesons observables (masses, decays, life-times) might carry relevant information related to possible restoration of symmetries under extreme conditions (quark-gluon plasma);

2) Both $\chi\, S$ and $U_A(1)\, S$ are restored or only $\chi\, S$?

- π^0:
  - non strange meson ;
  - best "Goldstone – like" boson;
  - mass due to explicit breaking of $\chi\, S$;

- η:
  - % of strangeness;
  - mass due to explicit breaking of $\chi\, S$ and $U_A(1)\, S$;

3) Will there be suppression or enhancement of those decays in medium?

Model and formalism:

• $SU(3)$ NJL model with 't Hooft determinant.

The Lagragian is:

$$\mathcal{L} = \bar{q}\,(i\partial\cdot\gamma - \hat{m})\,q + \frac{g_S}{2}\sum_{a=0}^{8}[(\bar{q}\lambda^a q)^2 + (\bar{q}(i\gamma_5)\lambda^a q)^2] + g_D[\det[\bar{q}(1-\gamma_5)q] + \det[\bar{q}(1-\gamma_5)q]]\,. \quad (1)$$

which can be rewritten as:

$$\mathcal{L} = \bar{q}\,(i\partial\cdot\gamma - \hat{m})\,q + \frac{1}{2}\{(\bar{q}\lambda^a q)S_{ab}(\bar{q}\lambda^b q) + (\bar{q}(i\gamma_5)\lambda^a q)P_{ab}(\bar{q}(i\gamma_5)\lambda^b q)\}\,, \quad (2)$$

where:

$$S_{ab} = g_S\delta_{ab} + g_D D_{abc} < \bar{q}\lambda^c q >\,, \quad (3)$$

$$P_{ab} = g_S\delta_{ab} - g_D D_{abc} < \bar{q}\lambda^c q >\,. \quad (4)$$

The hadronization procedure is making via the functional integral

$$Z \sim \int dq d\bar{q}\,\exp\{iW_{eff}[q,\bar{q}]\}\,, \quad (5)$$

where the quark effective action is:

$$W_{eff}[\varphi,\sigma] = -\frac{1}{2}\left(\sigma^a S_{ab}^{-1}\sigma^b\right) - \frac{1}{2}\left(\varphi^a P_{ab}^{-1}\varphi^b\right) - i\mathrm{Tr}\ln[i(\gamma_\mu\partial_\mu) - \bar{m} + \sigma_a\lambda^a + (i\gamma_5)(\varphi_a\lambda^a)]\,. \quad (6)$$

The first variation of the action $\Longrightarrow$ **Gap Equations**

Meson effective action $\Longrightarrow$ **Meson Propagators, $g_{M\bar{q}q}$, $f_{M\bar{q}q}$,...**

- **P. Costa et al., EPL (02), P. Costa et al., PLB (03), P. Costa et al., PRC (04)**

• Parameters and results:

$\mathbf{\Lambda}$ [Mev]	$\mathbf{m_u = m_d}$ [Mev]	$\mathbf{m_s}$ [MeV]	$\mathbf{g_S \Lambda^2}$	$\mathbf{g_D \Lambda^5}$
602.3	5.5	140.7	1.835	-12.36

$\mathbf{m_\pi}$ [Mev]	$\mathbf{f_\pi}$ [MeV]	$\mathbf{m_K}$ [MeV]	$\mathbf{m_\eta}$ [MeV]	$\mathbf{m_{\eta'}}$ [MeV]	$\mathbf{M_u = M_d}$ [MeV]	$\mathbf{M_s}$ [MeV]
135.0	92.4	497.7	514.8	957.8	367.7	549.5

- **S.P. Klevansky et al., PRC (96), P. Costa et al., EPL (02), P. Costa et al., PRC (04)**

• Formalism at finite temperature and density:

Generalize the NJL model to the finite temperature and chemical potential case $\Longrightarrow$ applying the Matsubara technique:

$$\int \frac{d^4p}{(2\pi)^4} \longrightarrow \frac{1}{-i\beta} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \sum_n , \qquad (7)$$

- $\beta = 1/T$ ($T \rightarrow$ **temperature**);
- $\omega_n = (2n+1)\frac{\pi}{\beta}$, $n = 0, \pm 1, \pm 2, ...,\rightarrow$ **Matsubara frequencies**;
- $k_0 \longrightarrow i\omega_n + \mu$ ($\mu \rightarrow$ **chemical potential**);
- **Integration over** $k_0 \rightarrow$ **sum over Matsubara frequencies**.

Gap equations:

$$\boxed{M_i = m_i - 2\,g_S << \bar{q}_i\,q_i >> - 2\,g_D << \bar{q}_j\,q_j >><< \bar{q}_k\,q_k >>} \quad (8)$$

with

$$<< \bar{q}_i\,q_i >> = 4I_1^i(T,\mu_i) \quad (9)$$

$$\Downarrow$$

quark condensates at finite T and μ_i;

- $$I_1^i(T,\mu_i) = -\frac{N_c}{4\pi^2}\int \frac{\mathtt{k}^2 d\mathtt{k}}{E_i}\left(n_i^+ - n_i^-\right); \quad (10)$$

- $$I_2^{ii}(P_0,T,\mu_i) = -\frac{N_c}{2\pi^2}\mathcal{P}\int \frac{\mathtt{k}^2 d\mathtt{k}}{E_i}\,\frac{1}{P_0^2 - 4E_i^2}\left(n_i^+ - n_i^-\right); \quad (11)$$

where

- $$n_i^{\mp} = n_i^{\mp}(E_i) = \frac{1}{1 + e^{\pm(E_i \pm \mu_i)}} \quad (12)$$

  are the Fermi distribution functions;

- $E_i = (k^2 + M_i^2)^{1/2}$;

- $M_i \rightarrow$ **mass of the constituent quarks**.

• Asymmetric quark matter with strange quarks in β equilibrium:

• β equilibrium:

$$\mu_d = \mu_s = \mu_u + \mu_e \tag{13}$$

• charge neutrality:

$$\frac{2}{3}\rho_u - \frac{1}{3}(\rho_d + \rho_s) - \rho_e = 0, \tag{14}$$

• quark particle number density:

$$\rho_i = \frac{1}{\pi^2}(\mu_i^2 - M_i^2)^{3/2}\theta(\mu_i^2 - M_i^2), \tag{15}$$

• electron particle number density:

$$\rho_e = \frac{\mu_e^3}{3\pi^2}, \tag{16}$$

• baryon particle number density:

$$\rho_B = \frac{1}{3}(\rho_u + \rho_d + \rho_s). \tag{17}$$

π^0 *propagator:*

The mass of the π^0 meson can be determined via the condition

$$\boxed{\left(1 - K_P\, J_{uu}(P_0 = m_{\pi^0}, \mathbf{P} = 0)\right) = 0} \tag{18}$$

with:

- $$K_P = g_S + g_D << \bar{s}s >>; \tag{19}$$

- $$J_{ii}(P_0) = 4(2I_1^i(T,\mu_i) + P_0^2 I_2^{ii}(P_0,T,\mu_i))\,; \tag{20}$$

- **Quark–pion coupling constant:**

$$g_{\pi^0\overline{q}q}^{-2} = -\frac{1}{2m_{\pi^0}}\frac{\partial}{\partial P_0}[J_{uu}(P_0)]_{|P_0=m_{\pi^0}}. \tag{21}$$

η *propagator:*

The mass of the η meson can be determined via the condition

$$D_\eta^{-1}(m_\eta, \mathbf{0}) = 0\,, \tag{22}$$

where

$$\boxed{D_\eta^{-1} = (\mathcal{A}+\mathcal{C}) - \sqrt{(\mathcal{C}-\mathcal{A})^2 + 4\mathcal{B}^2}} \tag{23}$$

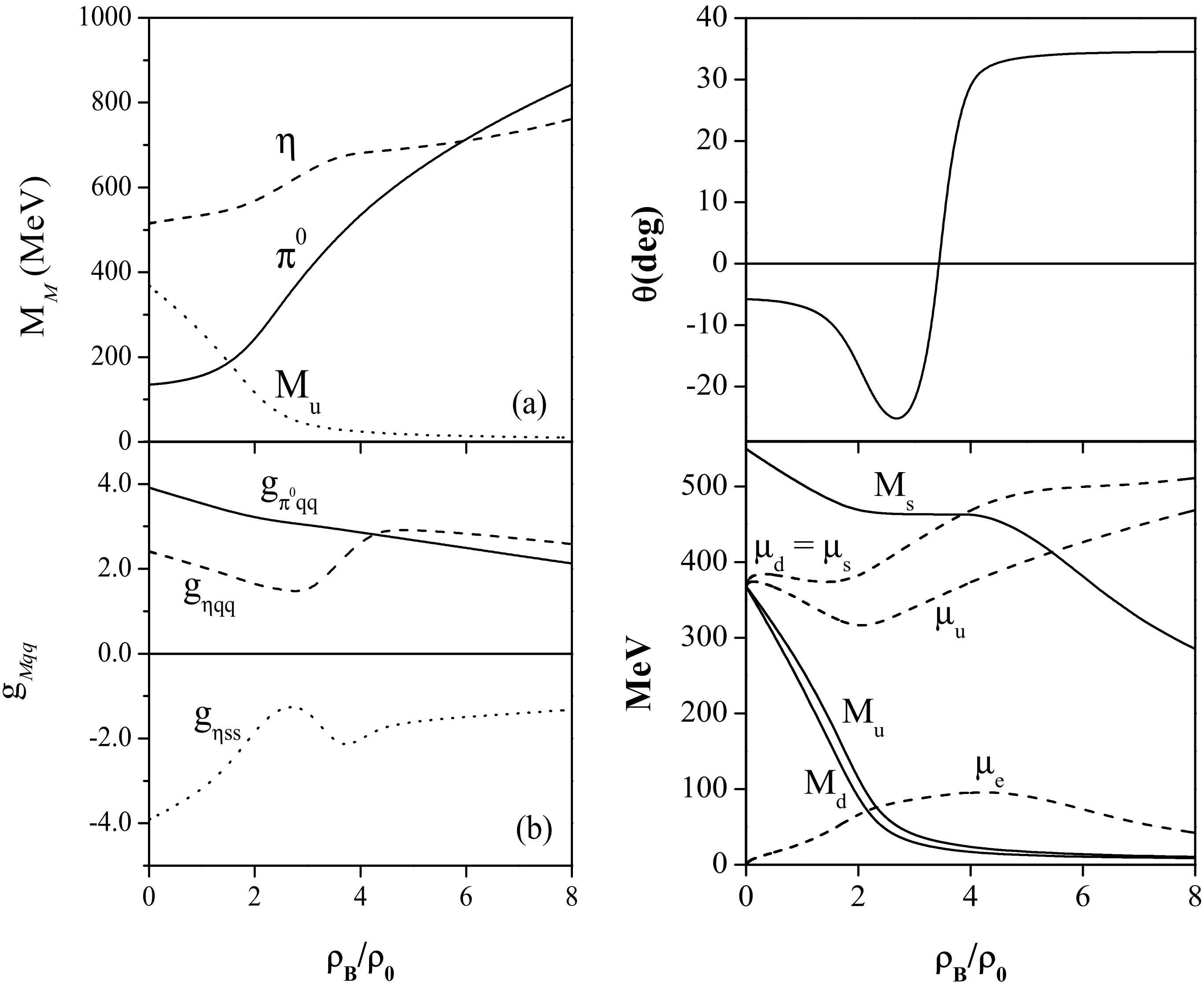
with

- $\begin{cases} \mathcal{A} = P_{88} - \Delta\Pi_{00}; \\ \mathcal{B} = -P_{08} - \Delta\Pi_{08}; \\ \mathcal{C} = P_{00} - \Delta\Pi_{88}; \\ \Delta = P_{00}P_{88} - P_{08}^2; \end{cases}$

- $\begin{cases} P_{00} = g_S - \frac{2}{3} g_D \left(<< \bar{q}_u\, q_u >> + << \bar{q}_d\, q_d >> + << \bar{q}_s\, q_s >>\right); \\ P_{08} = \frac{1}{3\sqrt{2}} g_D \left(<< \bar{q}_u\, q_u >> + << \bar{q}_d\, q_d >> -2 << \bar{q}_s\, q_s >>\right); \\ P_{88} = g_S + \frac{1}{3} g_D \left(2 << \bar{q}_u\, q_u >> +2 << \bar{q}_d\, q_d >> - << \bar{q}_s\, q_s >>\right); \end{cases}$

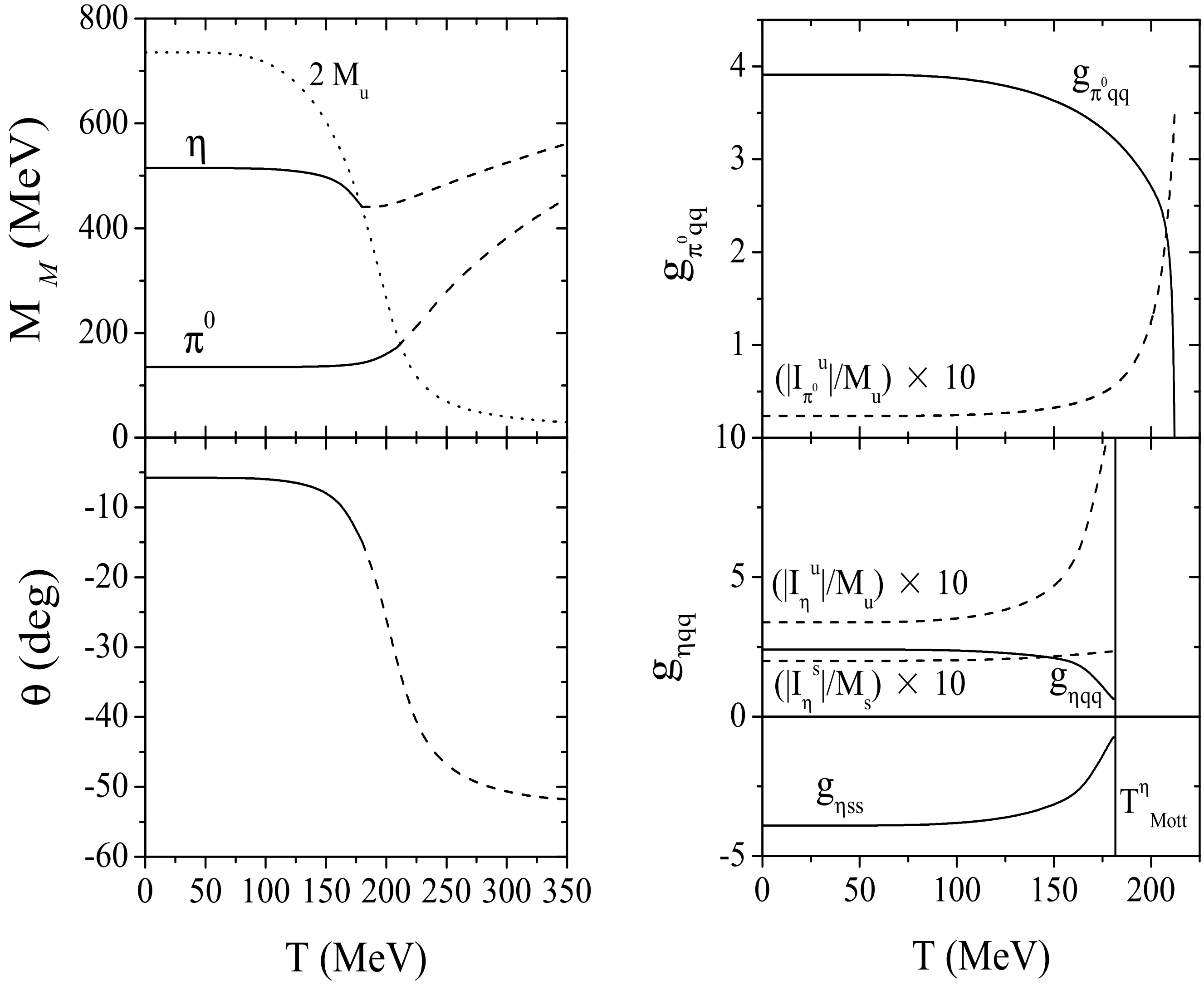
- $\begin{cases} \Pi_{00} = \frac{2}{3} \left(I_{uu}(m^2) + I_{dd}(m^2) + I_{ss}(m^2)\right); \\ \Pi_{08} = \frac{\sqrt{2}}{3} \left(I_{uu}(m^2) + I_{dd}(m^2) - 2I_{ss}(m^2)\right); \\ \Pi_{88} = \frac{1}{3} \left(I_{uu}(m^2) + I_{dd}(m^2) + 4I_{ss}(m^2)\right); \end{cases}$

• $T = 0,\ \rho \neq 0:$



Left Panel: a) Mesonic masses as function of the baryonic density; b) Meson-quark-quark coupling constant.
Right Panel: up – mixing angle θ; down – Constituent quark masses and chemical potentials as function of density.

• $\rho = 0$, $T \neq 0$:



Left Panel: Mesonic and quark masses (upper panel) and the mixing angle θ (lower panel) as functions of temperature.
Right Panel: meson quark coupling constants as functions of temperature.

The decay $M \to \gamma\gamma$

- $M \to \pi^0,\ \eta$

The transition amplitude of the decay $M \to \gamma\gamma$ is described by the two triangle diagrams

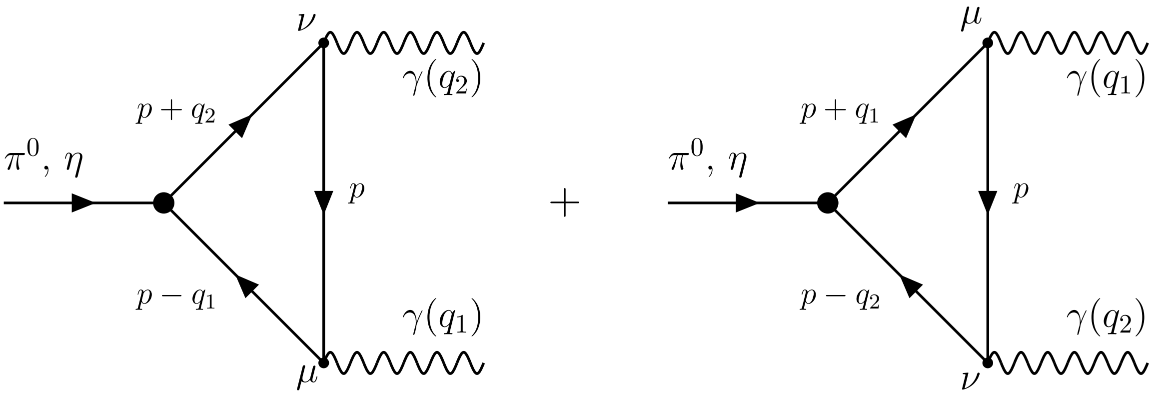


Figure 1: The quark triangle diagram for the η, $\pi^0 \to \gamma\gamma$ (direct and exchange process).

and reads:

$$\begin{aligned}\tilde{\mathcal{T}}_{M\to\gamma\gamma}(P,q_1,q_2) &= i\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left\{\Gamma_M S(p-q_1)\hat{\epsilon}_1 S(p)\hat{\epsilon}_2 S(p+q_2)\right\}\\ &+ \text{ exchange.}\end{aligned} \tag{24}$$

- $\hat{\epsilon}_{1,2}$ **photon polarization vector with momentum** $q_{1,2}$**;**
- $\mathbf{Tr} = \mathbf{tr}_c\mathbf{tr}_f\mathbf{tr}_\gamma$**;**
- **vertex function of the** $M-$**meson:** $\Gamma^M = i\gamma_5 g_{M\bar{q}q}$**.**

Using the kinematics:

$$P = q_1 + q_2, \tag{25}$$
$$P = (m_M, \mathbf{0}). \tag{26}$$

Transition amplitude:

- π^0 :

$$\boxed{|\mathcal{T}_{\pi^0\to\gamma\gamma}|^2 = (32\pi\alpha)^2 |g_{\pi^0\bar{q}q} I^u_{\pi^0}|^2}. \tag{27}$$

- η :

$$\boxed{\begin{array}{c} |\mathcal{T}_{\eta\to\gamma\gamma}|^2 = \frac{(32\pi\alpha)^2}{27} |\cos\theta(5 g_{\eta\bar{q}q} I^u_\eta - 2 g_{\eta\bar{s}s} I^s_\eta) \\ -\sqrt{2}\sin\theta(5 g_{\eta\bar{q}q} I^u_\eta + g_{\eta\bar{s}s} I^s_\eta)|^2. \end{array}} \tag{28}$$

with

$$I^i_M(P) = i M_i \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - M_i^2)[(p-q_1)^2 - M_i^2][(p+q_2)^2 - M_i^2]}. \tag{29}$$

Applying the Matsubara technique:

$$I^i_M(P_0, \mathbf{P}=0) = -\frac{M_i}{4\pi^2} \int_0^\infty dp \frac{p}{E_i^2} \frac{1}{4E_i^2 - P_0^2} \ln\left(\frac{E_i + p}{M_i}\right) \left[n_i^+ - n_i^-\right]. \tag{30}$$

The decay width:

$$\boxed{\Gamma_{M\to\gamma\gamma} = \frac{m_M^3}{64\pi} |\mathcal{T}_{M\to\gamma\gamma}|^2}. \tag{31}$$

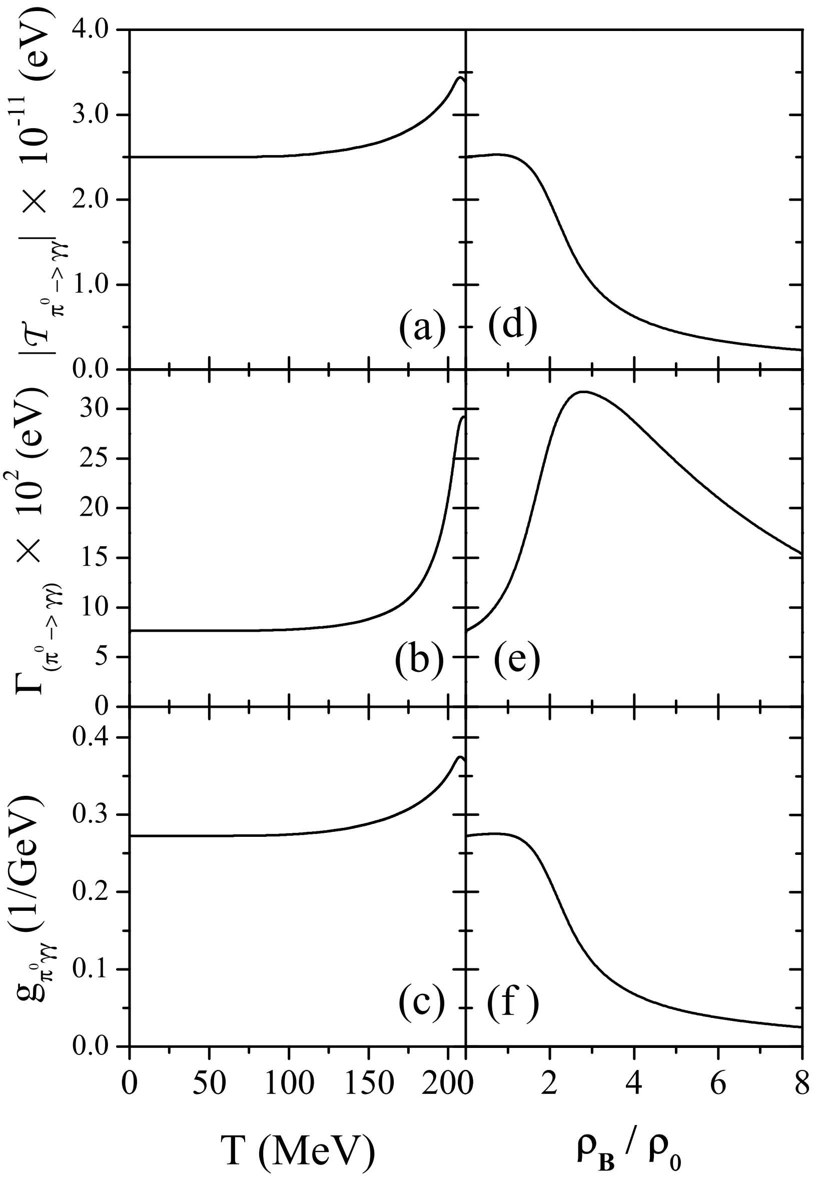
The decay coupling constant:

$$\boxed{g_{M\to\gamma\gamma} = \frac{\mathcal{T}_{M\to\gamma\gamma}}{e^2}}. \tag{32}$$

Numerical results at $T = \rho = 0$:

		NJL	Exp.
π^0	$\lvert \mathcal{T}_{\pi^0 -> \gamma\gamma} \rvert$ $[\mathrm{eV}]^{-1}$	2.5×10^{-11}	$(2.5 \pm 0.1) \times 10^{-11}$
	$\Gamma_{\pi^0 -> \gamma\gamma}$ [eV]	7.65	7.78(56)
	$g_{\pi^0\gamma\gamma}$ $[\mathrm{GeV}]^{-1}$	0.273	0.275
η	$\lvert \mathcal{T}_{\eta -> \gamma\gamma} \rvert$ $[\mathrm{eV}]^{-1}$	2.54×10^{-11}	$(2.5 \pm 0.06) \times 10^{-11}$
	$\Gamma_{\eta -> \gamma\gamma}$ [KeV]	0.440	0.465
	$g_{\eta\gamma\gamma}$ $[\mathrm{GeV}]^{-1}$	0.278	0.260

- π^0 :

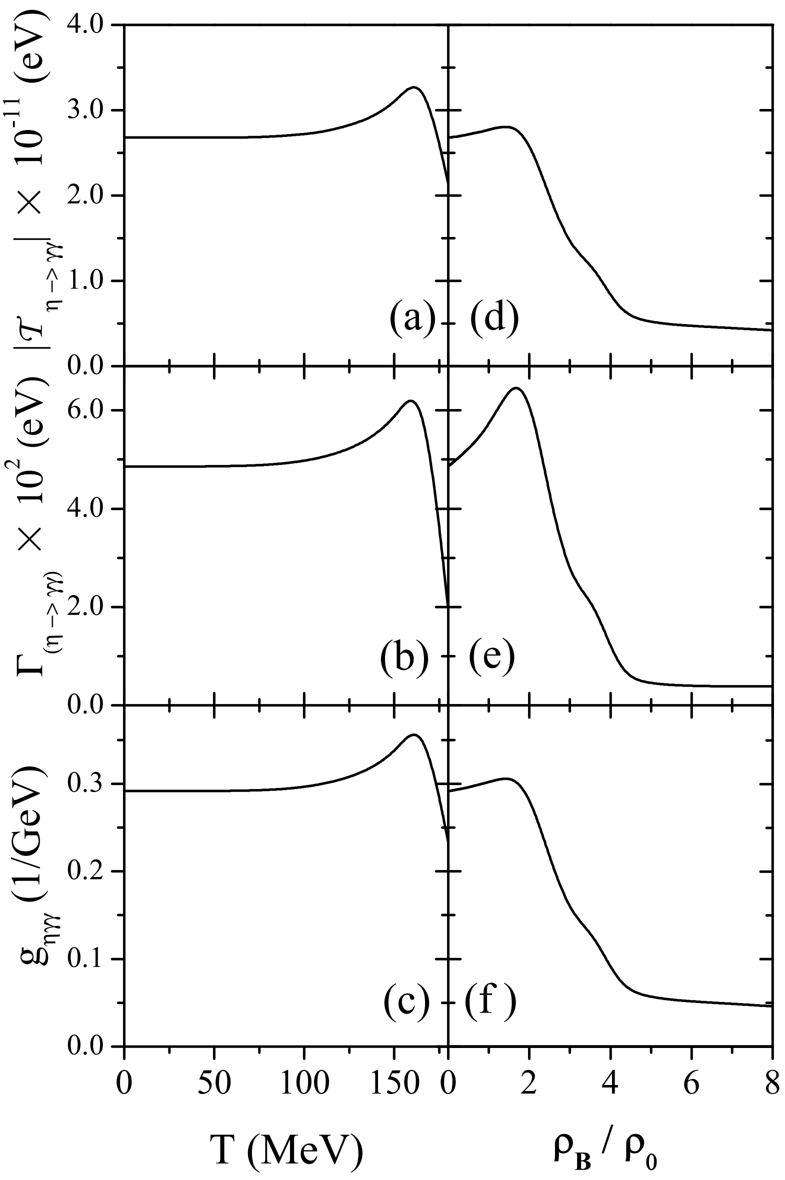


Transition amplitude, decay width and $\pi^0 \to \gamma\gamma$ coupling constant as functions of temperature and baryonic density.

- **Behavior of π^0 at finite T and ρ essentially reflects the restoration of $SU(2)$ χS:**
  - $T = 0$**,** $\rho \neq 0$:
    1. $m_{\pi^0} \approx m_\sigma \longrightarrow$ as the density increases;
    2. $f_{\pi^0} \longrightarrow 0$;
    3. $g_{\pi^0 \bar{q}q} \longrightarrow$ decreases (π^0 less coupled to quarks)
  - $\rho = 0$**,** $T \neq 0$:
    1. $m_{\pi^0} \approx m_\sigma \longrightarrow$ as the temperature increases;
    2. $T^{\pi^0}_{Mott} \approx 212$ MeV

- $\pi^0 \longrightarrow \gamma\gamma$ **observables:**
  - $T = 0$**,** $\rho \neq 0$:
    1. M_u and $g_{\pi^0 \bar{q}q}$ decrease with density $\Rightarrow$ $\mathcal{T}_{\pi^0 \to \gamma\gamma}$ and $g_{\pi^0 \to \gamma\gamma}$ decrease.
    2. $\Gamma_{\pi^0 \to \gamma\gamma}$:
       * $\rho \simeq 2.5\rho_0 \longrightarrow$ maximum;
       * two competitive effects: the decrease of the transition amplitude and, the increase of the π^0 mass;
       * $\rho > 2.5\rho_0 \longrightarrow$ the two photon decay of the pion becomes less favorable.
  - $\rho = 0$**,** $T \neq 0$:
    1. two competitive effects: The decrease of $g_{\pi^0 \bar{q}q}$ and the increase of $I^u_{\pi^0}$ $\Rightarrow$ $\mathcal{T}_{\pi^0 \to \gamma\gamma}$ and $g_{\pi^0 \to \gamma\gamma}$ increase smoothly until $T^{\pi^0}_{Mott}$.
    2. $\Gamma_{\pi^0 \to \gamma\gamma} \Rightarrow$ sharp increase due M_{π^0} increasing.

- η :



Transition amplitude, decay width and $\eta \to \gamma\gamma$ coupling constant as functions of temperature and baryonic density.

- **Behavior of η at finite T and ρ in β equilibrium reflects:**
  - $T = 0$, $\rho \neq 0$:
    1. restoration of χS in strange and non strange sectors;
    2. the evolution of η quark structure:

$$
\begin{aligned}
|\eta > &= \cos\theta \frac{1}{\sqrt{3}} |\bar{u}u + \bar{d}d - 2\bar{s}s > \\
&- \sin\theta \sqrt{\frac{2}{3}} |\bar{u}u + \bar{d}d + \bar{s}s > .
\end{aligned}
\tag{33}
$$

    $*$ $\rho > 3.8\rho_0 \Rightarrow$ existence of strange valence quarks;
    $*$ $\rho \simeq 3.5\rho_0$ mixing angle θ changes sign;
    $*$ $\rho > 3.8\rho_0$ η is more strange.
  - $\rho = 0$, $T \neq 0$:
    1. $T^{\eta}_{Mott} \approx 180$ MeV

- **Behavior of $\eta \to \gamma\gamma$ observables:**
  - Qualitatively similar to $\pi^0 \to \gamma\gamma$ at $T \neq 0$ and $\rho \neq 0$.

Summary:

- **Good agreement with data at $T = 0$, $\rho = 0$;**

- **In spite of the different structure of these mesons, π^0 and η decays exhibit a qualitatively similar behavior at finite T and ρ;**

- **This behavior mainly reflects the restoration of chiral symmetry χS;**

- **The anomalous decays of $\pi^0(\eta) \to \gamma\gamma$ are significantly affected by the medium;**

- **The enhancement of width is probably not sufficient to lead to lifetimes shorter than the expected lifetime of the fireball $\Rightarrow$ The decays are probably observed only after freeze-out: $\tau_{\pi^0\to\gamma\gamma} \approx 10^{-17}$ s; $\tau_{\eta\to\gamma\gamma} \approx 10^{-18}$ s $>> \tau_{fireball} \approx 10^{-22}$ s .**