

Gapless phases of color superconducting matter

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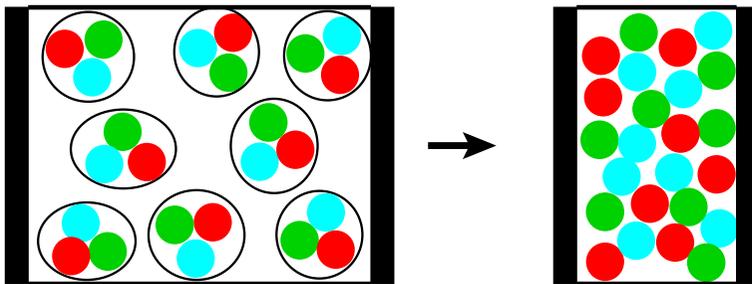
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References

- I. Shovkovy and M. Huang, Phys. Lett. B **564** (2003) 205, hep-ph/0302142
- M. Huang and I. Shovkovy, Nucl. Phys. A **729** (2003) 835, hep-ph/0307273
- S. Rüster, I. Shovkovy, and D. Rischke, Nucl. Phys. A **743** (2004) 127, hep-ph/0405170
- M. Huang and I. Shovkovy, hep-ph/0407049; hep-ph/0408268

Very dense baryonic matter

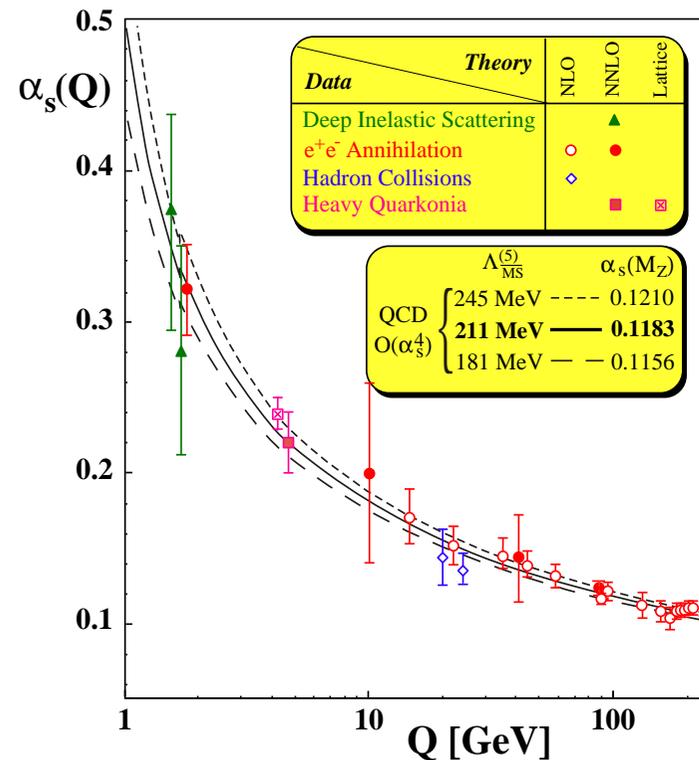
- “Squeezing” baryonic matter hard should produce quark matter:



- Asymptotic freedom: $\alpha_s(\mu) \ll 1$
 $\mu \gg \Lambda_{QCD}$ [Gross&Wilczek; Politzer,'73]
- Very dense quark matter is **weakly** interacting [Collins&Perry,'75]

☹ Note: realistic densities in stars are not sufficiently large:

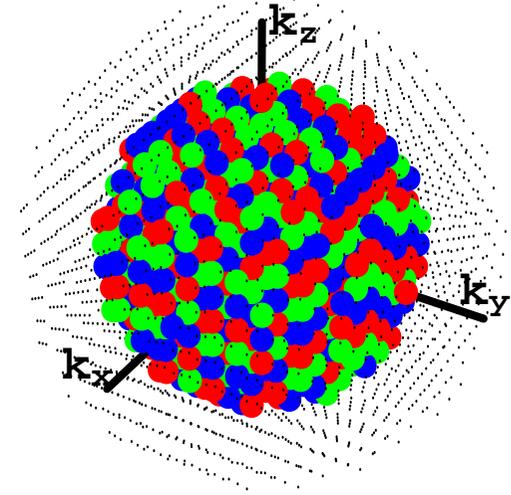
$$\rho \lesssim 10\rho_0, \text{ where } \rho_0 \approx 0.15 \text{ fm}^{-3} \quad \Rightarrow \quad \mu \lesssim 500 \text{ MeV}$$



Ground state of dense quark matter

Educated guess:

- (i) Quarks are fermions ($s = \frac{1}{2}$)
(ii) Interaction is weak ($\alpha_s \ll 1$) } \Rightarrow Fermi liquid (?)
(cf., electron gas in metals/alloys)



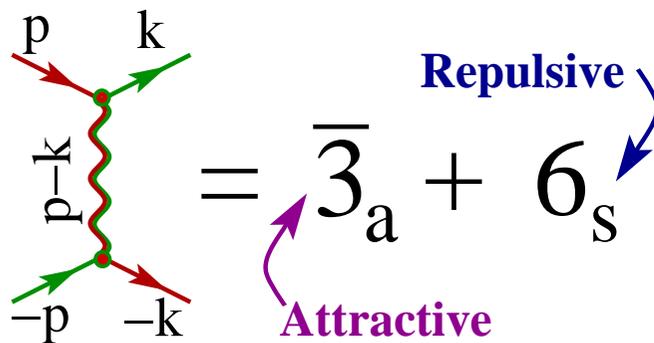
Further refinement:

- (i) Degenerate Fermi surface
(ii) Attractive interaction (?) } \Rightarrow Cooper instability
(cf., the Cooper instability in superconducting metals/alloys)

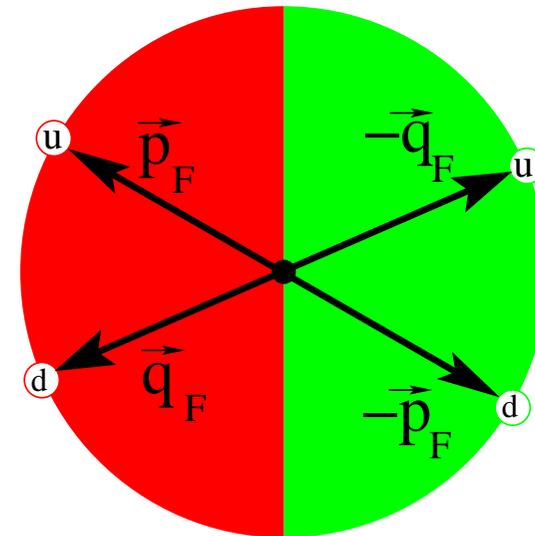
Color superconductivity in dense QCD

Strange quark matter:

- $N_f = 3$: u-, d- and s-quarks
- $N_c = 3$: “red”, “green” and “blue”
- $k_F^{(u)} = k_F^{(d)} = k_F^{(s)} = \mu$
- Quark-quark interaction:



$$\langle \mathbf{u}_p \mathbf{d}_{-p} \rangle = - \langle \mathbf{u}_q \mathbf{d}_{-q} \rangle \neq 0$$



Cooper instability \rightarrow color superconductivity

$$(|\bullet\bullet\rangle - |\bullet\bullet\rangle)_{\bar{3}} \otimes (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_0 \otimes (|u,d\rangle - |d,u\rangle)_{\bar{3}} \quad (\Leftarrow \text{Pauli principle})$$

Color-flavor locked phase

- Wave function of a Cooper-pair:

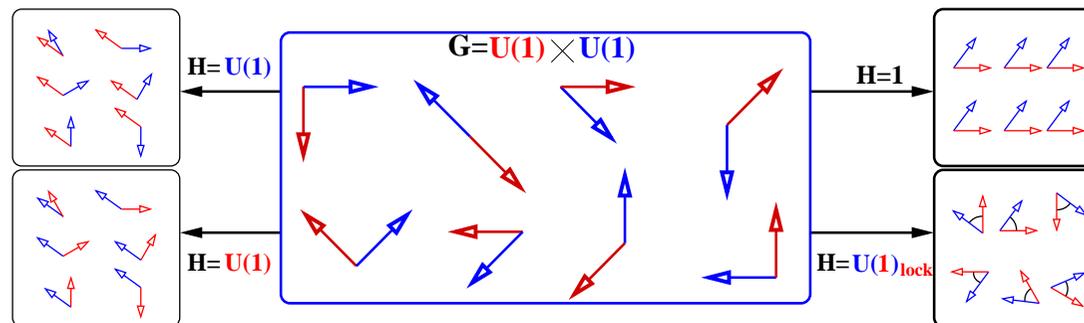
Pauli principle: $(|\bullet\bullet\rangle - |\bullet\bullet\rangle)_{\bar{3}} \otimes (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_{J=0} \otimes (|u,d\rangle - |d,u\rangle)_{\bar{3}}$

- Diquark condensate (spin-0 gap $\sim 10 - 100$ MeV):

$$\langle (\bar{\Psi}_L^C)_i^\alpha (\Psi_L)_j^\beta \rangle = \langle (\bar{\Psi}_R^C)_i^\alpha (\Psi_R)_j^\beta \rangle \simeq \sum \varepsilon^{\alpha\beta I} \epsilon_{ijI} \Delta$$

- Color-flavor locking: $SU(3)_L \times SU(3)_R \times SU(3)_c \rightarrow SU(3)_{L+R+c}$

[Alford et al. hep-ph/9804403]



There are no $\langle q_L q_R \rangle$ condensates, but chiral $SU(3)_L \times SU(3)_R$ symmetry is broken down to $SU(3)_V$ through locking with color!

Color superconductivity in stars

Is there CSC matter inside compact stars?

Arguments in favor:

- 😊 Relatively high densities in stars, $\rho_c \gtrsim 5\rho_0$, suggest that quarks may be deconfined
- 😊 An attractive diquark channel is likely to exist
- 😊 Temperatures are quite low, $T \lesssim 50$ keV, to allow pairing

Arguments against:

- 😞 Strongly coupled dynamics is not under control
- 😞 Matter may not necessarily be deconfined at existing densities
- 😞 Specific conditions inside stars (e.g., β -equilibrium) may not favor color superconductivity

The natural approach: to give model predictions and to test them

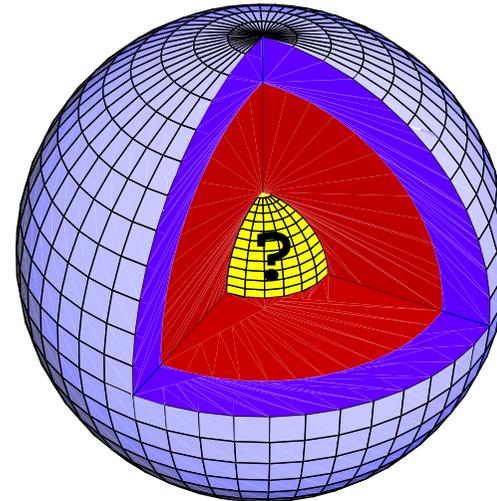
Importance of neutrality inside a star

Matter inside star should be

(i) β -equilibrated: $\mu_d = \mu_u + \mu_e$
 $\mu_s = \mu_u + \mu_e$

(ii) charge neutral:

$$n_Q^{\text{el}} = 0, \quad n_Q^{\text{color}} = 0$$



• Coulomb energy (when $n_Q \neq 0$)

$$E_{\text{Coulomb}} \sim n_Q^2 R^5 \sim M_{\odot} c^2 \left(\frac{n_Q}{10^{-15} e/\text{fm}^3} \right)^2 \left(\frac{R}{1 \text{ km}} \right)^5$$

In 2SC phase, $10^{-2} \lesssim n_Q \lesssim 10^{-1} e/\text{fm}^3 \Rightarrow E_{\text{Coulomb}}^{2\text{SC}} \gg M_{\odot} c^2$

Neutrality vs. $N_f = 2$ color superconductivity

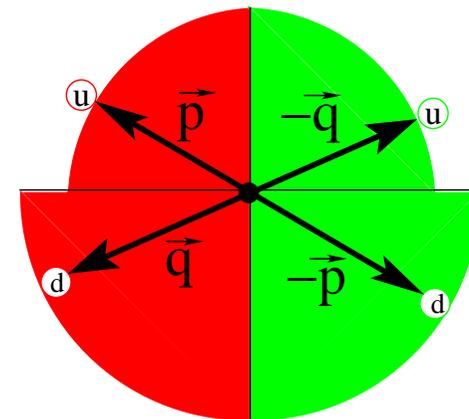
- The “best” 2SC phase appears when $n_d \approx n_u$
- Neutral matter (in β -equilibrium) appears when $n_d \approx 2n_u$
- Electrons do **not** help (!):

$$n_d \approx 2n_u \Rightarrow \mu_d \approx 2^{1/3} \mu_u \Rightarrow \mu_e = \mu_d - \mu_u \approx \frac{1}{4} \mu_u$$

$$\text{i.e.,} \quad n_e \approx \frac{1}{4^3} \frac{n_u}{3} \ll n_u$$

The “best” Cooper pairing is distorted by the following mismatch parameter:

$$\delta\mu \equiv \frac{p_F^{\text{down}} - p_F^{\text{up}}}{2} = \frac{\mu_e}{2} \neq 0$$



Appearance of a gapless phase

Mismatch parameter μ_e is **not** a free model parameter,

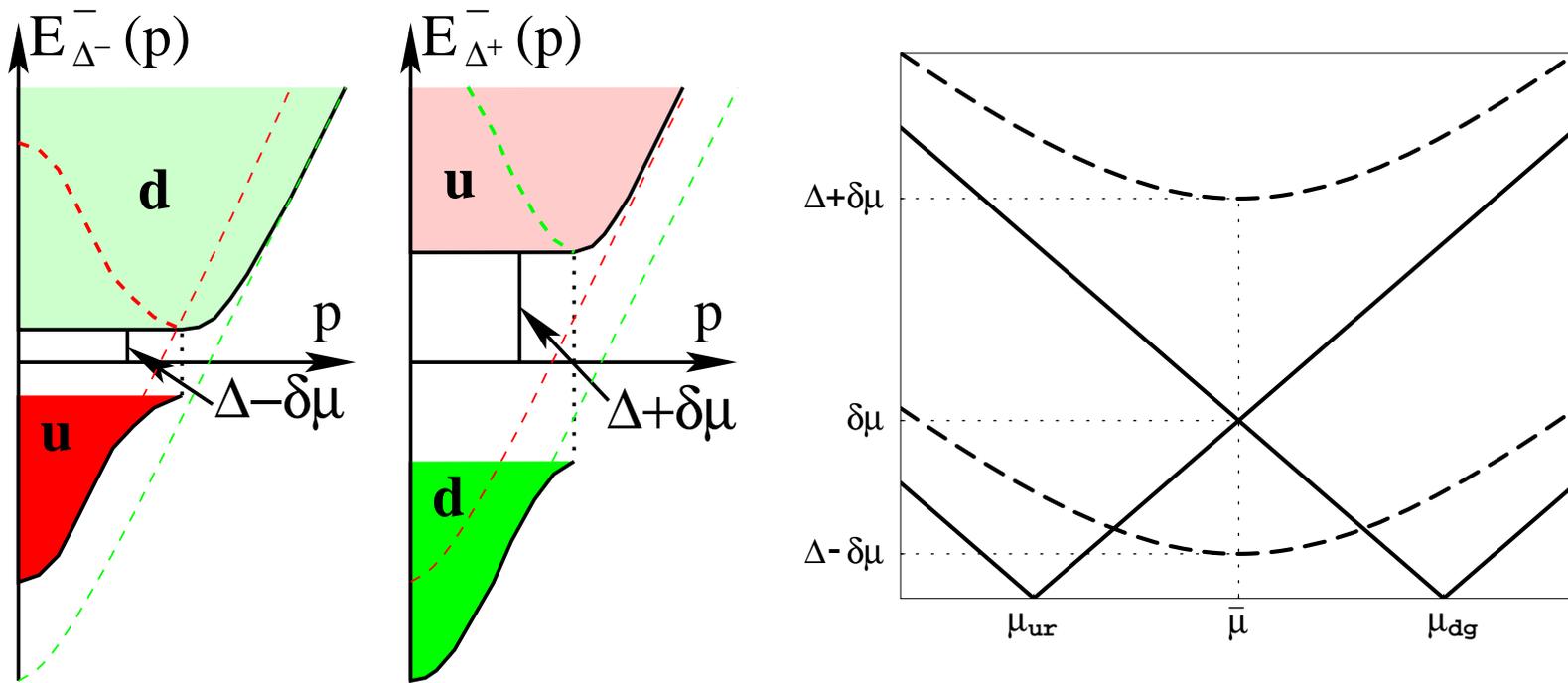
$$n_Q \equiv -\frac{\partial\Omega}{\partial\mu_e} = 0 \quad \Rightarrow \quad \mu_e = \mu_e(\mu, \Delta)$$

The diquark coupling strength (η) is a model parameter:

1. $\eta \lesssim 0.7$ — the mismatch does not allow Cooper pairing:
normal phase is the ground state
2. $\eta \gtrsim 0.8$ — strong coupling wins over the mismatch between the Fermi surfaces: 2SC is the ground state
3. $0.7 \lesssim \eta \lesssim 0.8$ — regime of intermediate coupling strength:
the ground state is the gapless 2SC phase [hep-ph/0302142]

Quasiparticle spectrum in 2SC phase

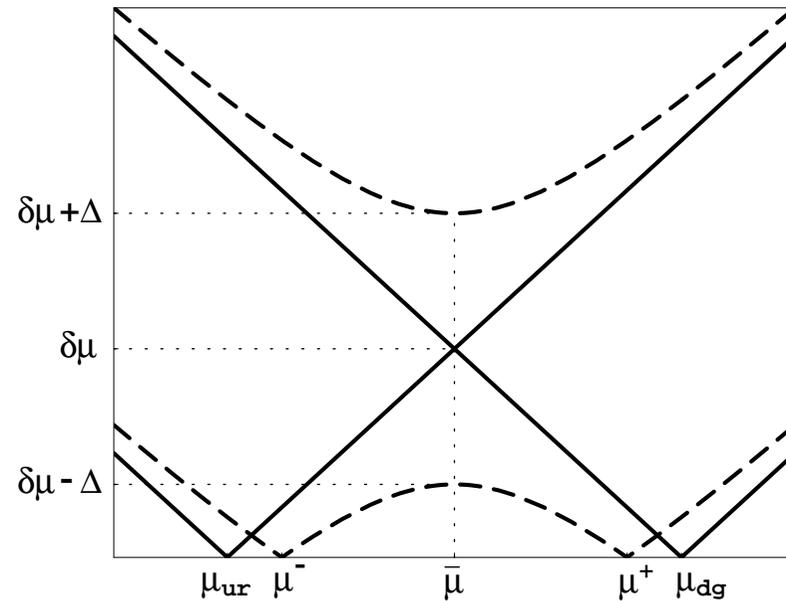
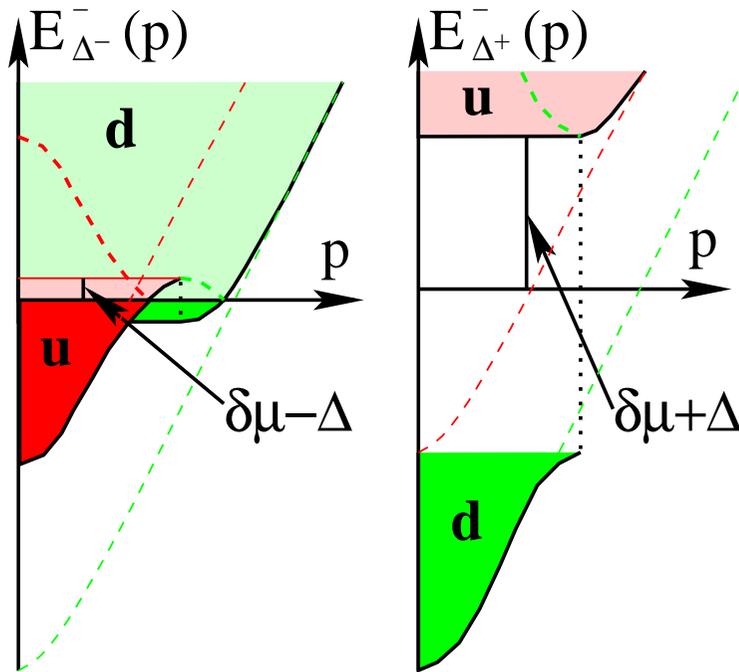
“Strong” coupling (2SC phase)



The energy gaps in the quasiparticle spectra are $\Delta - \delta\mu$ & $\Delta + \delta\mu$

Quasiparticle spectrum in g2SC phase

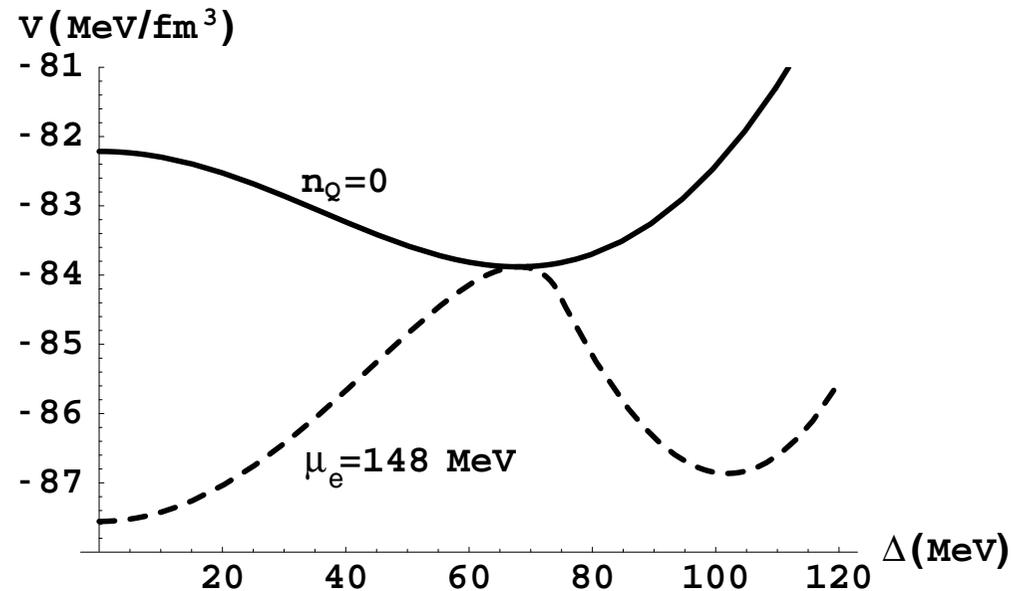
“Intermediate” coupling (gapless phase)



The energy gaps in the quasiparticle spectra are 0 & $\Delta + \delta\mu$

Thermodynamic stability of g2SC phase

Effective potential at $T = 0$ [I.S. & M.Huang, Phys. Lett. B564 (2003) 205]:



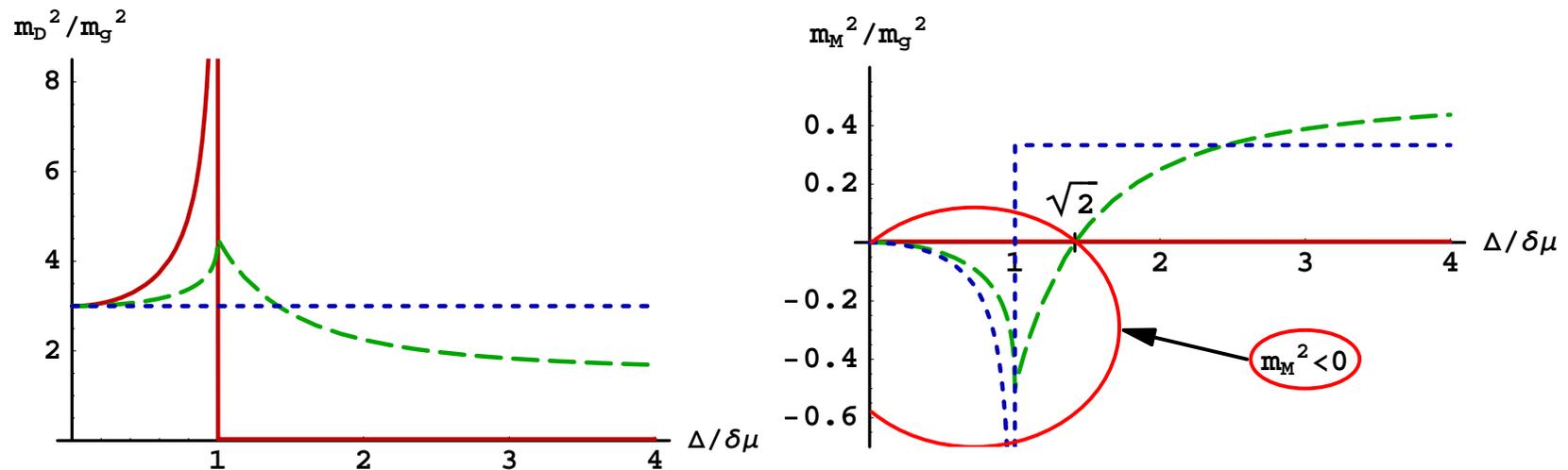
(Q.: Mixed phase? → A.: Unlikely if $\sigma \gtrsim 20 \text{ MeV}/\text{fm}^2$ [Shovkovy, Hanauske, Huang, hep-ph/0303027]. See, however, [Reddy & Rupak, nucl-th/0405054])

No Sarma instability in g2SC phase if $n_Q = 0$ is enforced *locally*!

Chromomagnetic instability

Recent results for gluon screening masses

[Huang & Shovkovy, hep-ph/0407049; hep-ph/0408268]:



- $A = 1, 2, 3$ — red solid line
- $A = 4, 5, 6, 7$ — green long-dash line
- $A = \tilde{8}$ — blue short-dash line

Finite strange quark mass, $0 < m_s < \infty$

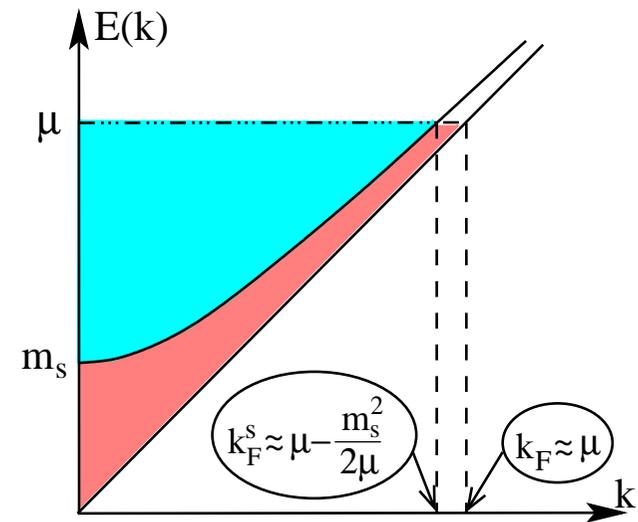
Fermi momentum of strange quarks is lowered:

$$k_F^{(s)} \simeq \mu - \frac{m_s^2}{2\mu}$$

The ground state of strange quark matter may have:

- only spin-1 condensates of same flavor
- only superconductivity of up and down quarks (2SC or g2SC)
- crystalline pairing (nonzero momentum pairing, LOFF)

Recently, other possibilities were proposed as well ...



Gapless CFL phase

- Mismatch parameter $\delta\mu \equiv m_s^2/2\mu = -\mu_8$

$$\mu_{bd}^{\text{eff}} = \mu - \frac{2}{3}\mu_8, \quad \text{and} \quad \mu_{gs}^{\text{eff}} = \mu + \frac{1}{3}\mu_8 - \frac{m_s^2}{2\mu}$$

- Gapless modes appear when $|\delta\mu| \equiv \frac{m_s^2}{2\mu} > \Delta_0$

$T = 0$: [Alford, Kouvaris & Rajagopal, hep-ph/0311286]

- More possibilities at $T \neq 0$

[Iida et al, hep-ph/0312363], [Rüster, Shovkovy & Rischke, hep-ph/0405170]

- Many different color-flavor pairing channels (3 or 9 gap functions)

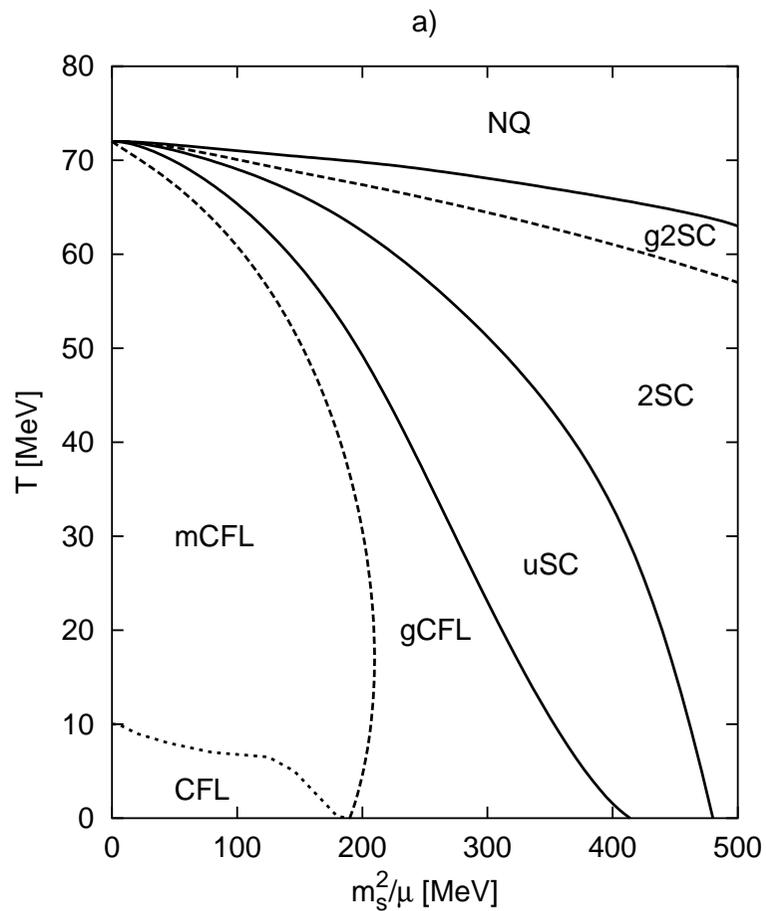
$$\Delta_{ij}^{\alpha\beta} \simeq \Delta_1 \epsilon_{1ij} \epsilon^{1\alpha\beta} + \Delta_2 \epsilon_{2ij} \epsilon^{2\alpha\beta} + \Delta_3 \epsilon_{3ij} \epsilon^{3\alpha\beta} + \dots$$

- Zoo of phases: CFL, mCFL, gCFL, uSC, dSC, 2SC, g2SC, NQ

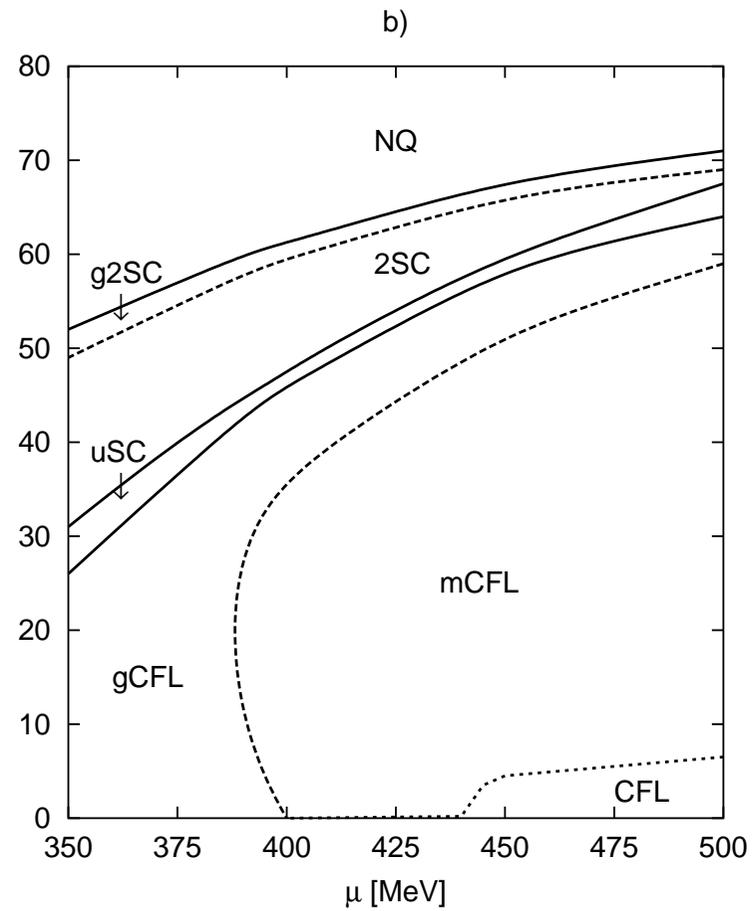
Overview of phases with strangeness

- Color-flavor locked (CFL) phase
 - “Enforced pairing”: $n_u = n_d = n_s$ ($T \simeq 0$)
[Rajagopal, Wilczek, 2001]
 - Natural insulator, $n_{el} \simeq 0$
 - Little specific heat and low neutrino emissivity
- Metallic CFL phase ($n_{el} \neq 0$)
 - $T = 0$: gapless CFL phase (no “enforced pairing”)
 - $T \neq 0$: thermal effects $\rightarrow n_{el} \neq 0$
 - Large specific heat and high neutrino emissivity
- uSC phase: only ud - & us -pairing (no ds -pairing)
- dSC phase: only du - & ds -pairing (no us -pairing)

Phase diagram



a) $\mu = 500$ MeV



b) $m_s = 250$ MeV

Current status

- At $\mu \gg \Lambda_{QCD}$, QCD dynamics is weakly coupled, but non-perturbative
- In this limit, QCD can be studied from first principles
- Some features of $T - \mu$ phase diagram start to develop
- In particular, sufficiently dense matter is a color superconductor
- Neutrality and β -equilibrium strongly affect the properties of CSC matter
- There can exist many different CSC phases (e.g., 1SC, 2SC, g2SC, CFL, gCFL, mCFL, uSC, dSC, LOFF, CFL+K⁰, CFL+ η)
- An intensive search for signature-type observables of CSC inside stars is under way

Outlook

- A systematic study of competition between different phases in dense QCD should be completed
- Physical properties (transport, in particular) of QCD phases should be addressed in detail
- The status of gapless phases should be resolved (addressing, e.g., the chromomagnetic instability, spontaneously induced currents)
- The most promising observable(s), (dis-)proving the presence of CSC inside stars, should be proposed
- A rigorous approach to treat QCD at nonzero densities should be developed