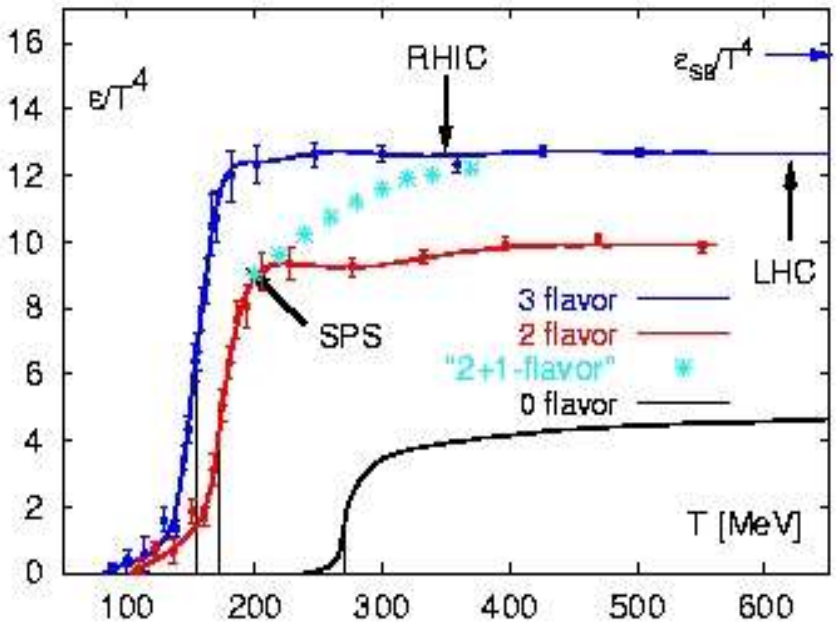
Chemical freeze-out and the QCD phase boundary

- Comments on the QCD phase transition
  - The chemical freeze-out curve
  - Analysis near $\mu = 0$
    - 2-body collisions don't equilibrate
    - the phase transition drives equilibration through multi-hadron collisions
  - Speculation about the phase boundary at large μ
  - Outlook

SQM2004, Capetown, South-Africa

Critical energy density and critical temperature



Lattice QCD calculations for $\mu_B = 0$
Karsch et al, hep-lat/0305025

$T_c = 173 \pm 12$ MeV

$\varepsilon_c = 700 \pm 200$ MeV/fm^3

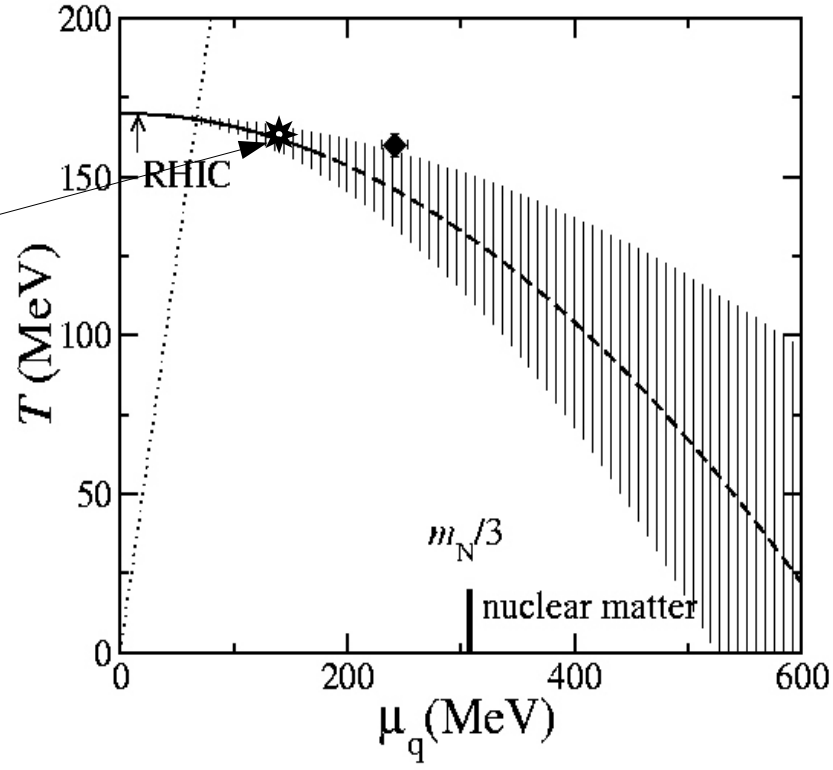
for the (2 + 1) flavor case:

the phase transition to the QGP and its parameters are quantitative predictions of QCD.

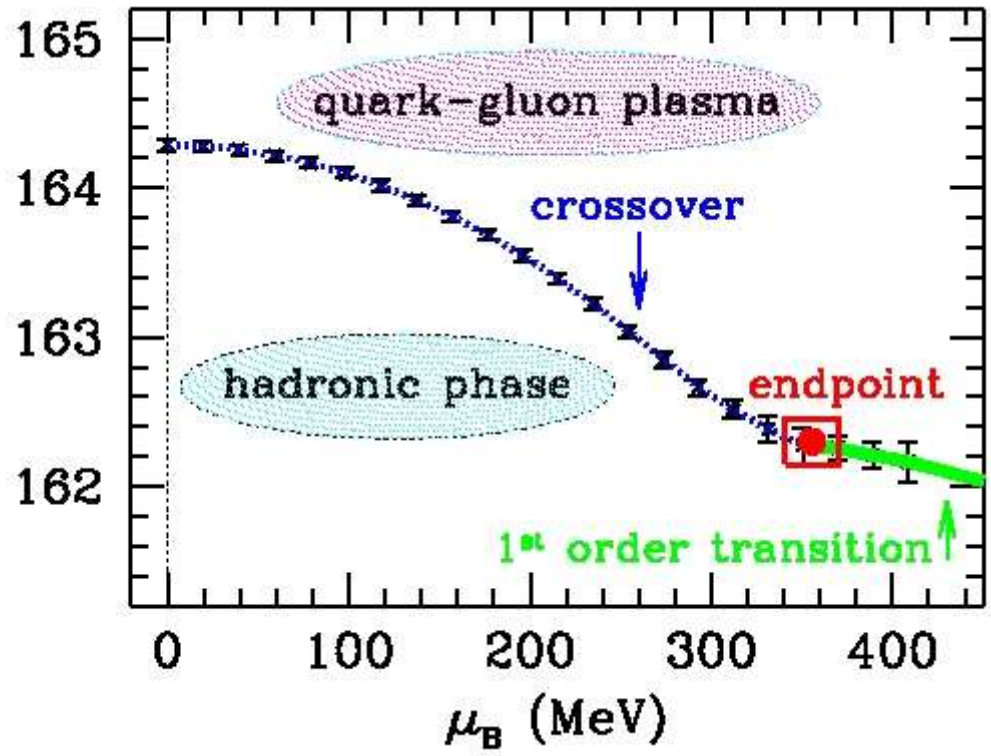
The order of the transition is not yet definitively determined.

The QCD phase boundary – recent results from lattice QCD

New endpoint



S. Ejiri et al, hep-lat/0312006



Z. Fodor, S. Katz, JHEP0404, (2004) 050;

Note: $3\,\mu_q = \mu_B$

Tri-critical point not (yet) well determined theoretically

Hadron yields signal chemical equilibrium

- From AGS energy on, all hadron yields in central PbPb collisions reflect grand-canonical equilibration
- Strangeness suppression observed in elementary collisions is lifted

For a recent review see:

pbm, Stachel, Redlich, QGP3, R. Hwa, editor, Singapore 2004, nucl-th/0304013

Thermal model description of hadron yields

HELMHOLTZ GEMEINSCHAFT

Grand Canonical Ensemble

$$\ln Z_i = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln(1 \pm \exp(-(E_i - \mu_i)/T))$$

$$n_i = N/V = -\frac{T}{V}\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2\,\mathrm{d}p}{\exp((E_i - \mu_i)/T) \pm 1}$$

$$\mu_i = \mu_B B_i + \mu_S S_i + \mu_{I_3} I_i^3$$

Fit at each energy provides values for T and μ_b

for every conserved quantum number there is a chemical potential μ
but can use conservation laws to constrain:

- Baryon number: $V \sum_i n_i B_i = Z + N \quad \rightarrow V$
- Strangeness: $V \sum_i n_i S_i = 0 \quad \rightarrow \mu_S$
- Charge: $V \sum_i n_i I_i^3 = \frac{Z - N}{2} \quad \rightarrow \mu_{I_3}$

This leaves only μ_b and T as free parameter when 4π considered

for rapidity slice fix volume e.g. by dN_{ch}/dy

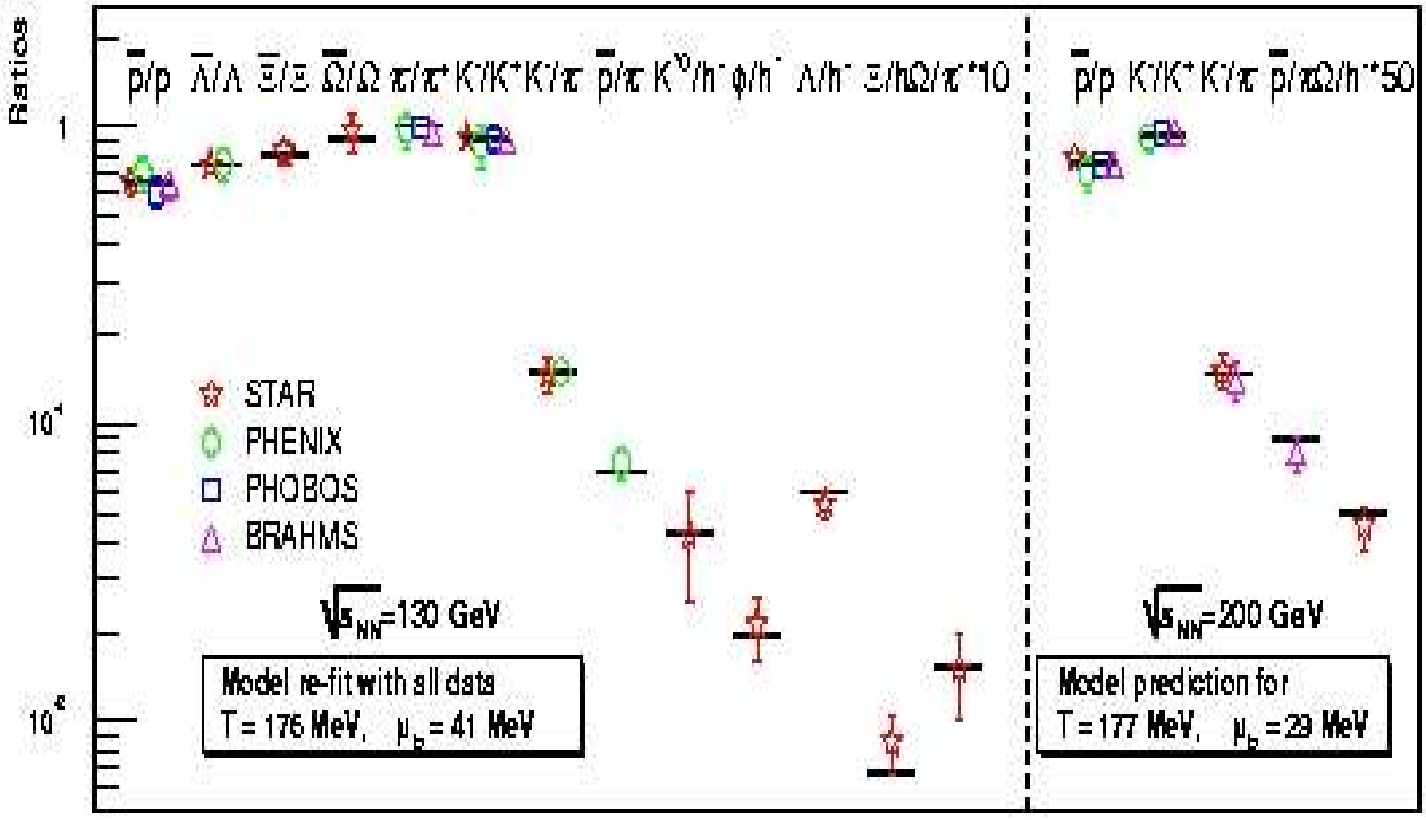
Hadro-chemistry at RHIC -- weakly decaying particles

All data in excellent agreement with thermal model predictions

chemical freeze-out at: T = 175 ± 8 MeV

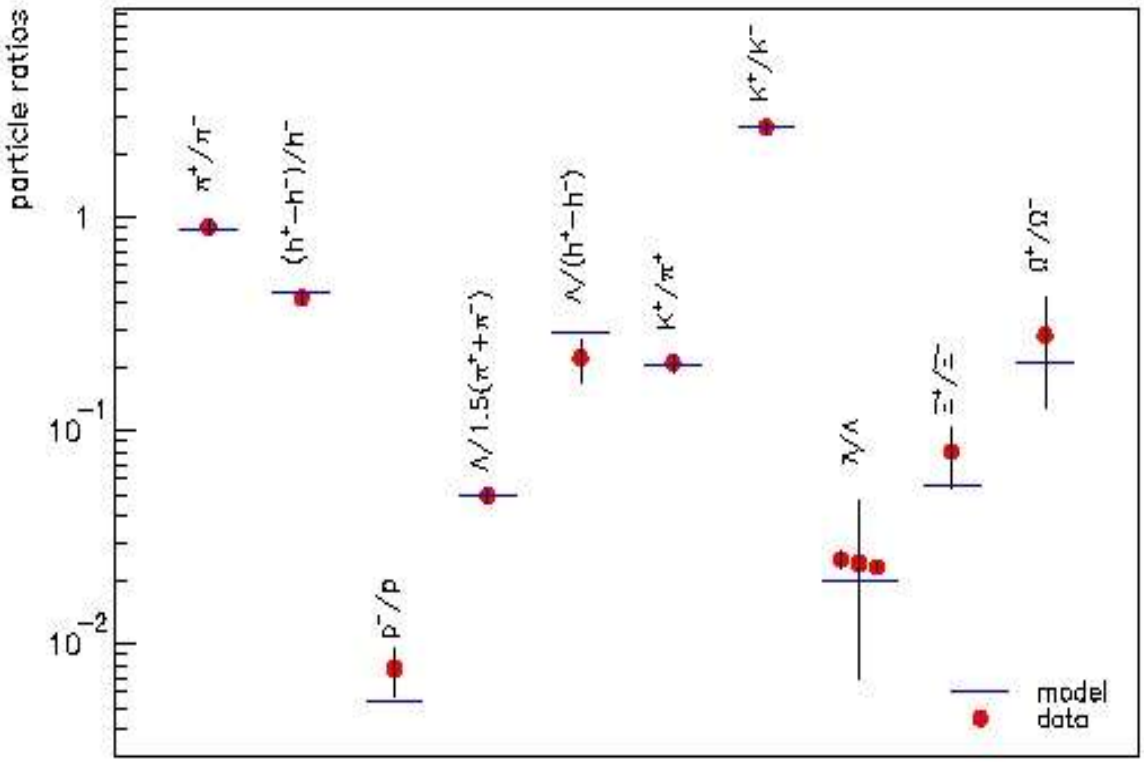
fit uses vacuum masses

new results from SQM04 consolidate this picture



pbm, d. magestro, j. stachel, k. redlich, Phys. Lett. B518 (2001) 41; see also Xu et al., Nucl. Phys. A698(2002) 306; Becattini, J. Phys. G28 (2002) 1553; Broniowski et al., nucl-th/0212052.

Hadro-chemistry at SPS



Data at 40 GeV/u Pb+Pb central collisions

T = 148 MeV,

μ_b= 400 MeV

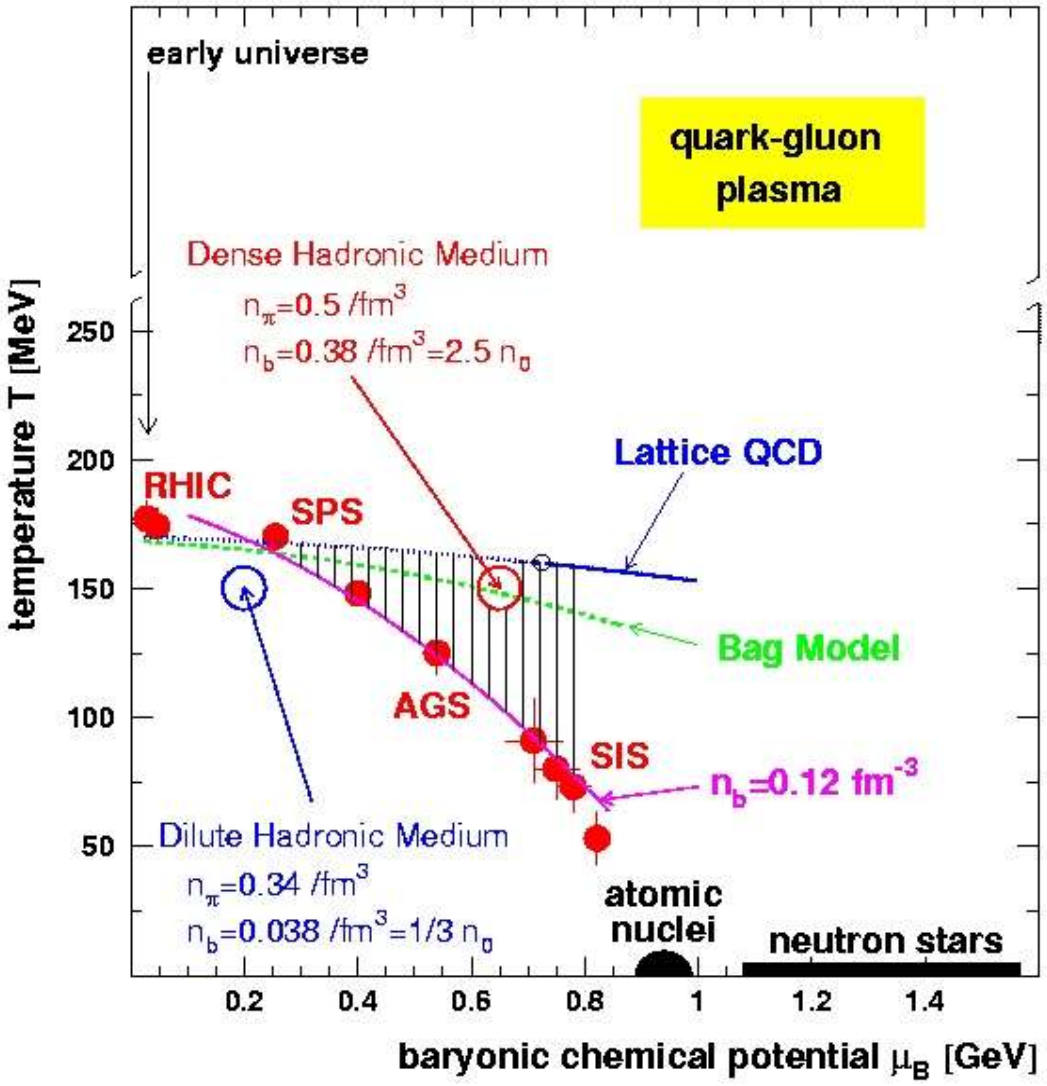
analysis from pbm, Stachel, Redlich, nucl-th/0304013 "Quark-gluon plasma 3, p. 491 – 599"

Chemical freeze-out curve – the view as of 2002

P. Braun-Munzinger, J. Stachel,
J. Phys. G. 28 (2002) 1971
chem. freeze-out at constant total baryon density

J. Cleymans, K. Redlich,
Phys. Rev. Lett. 81(1998)5284
chem. freeze-out at constant energy/particle

Note: for $\mu < 300$ MeV, phase boundary coincides with freeze-out curve

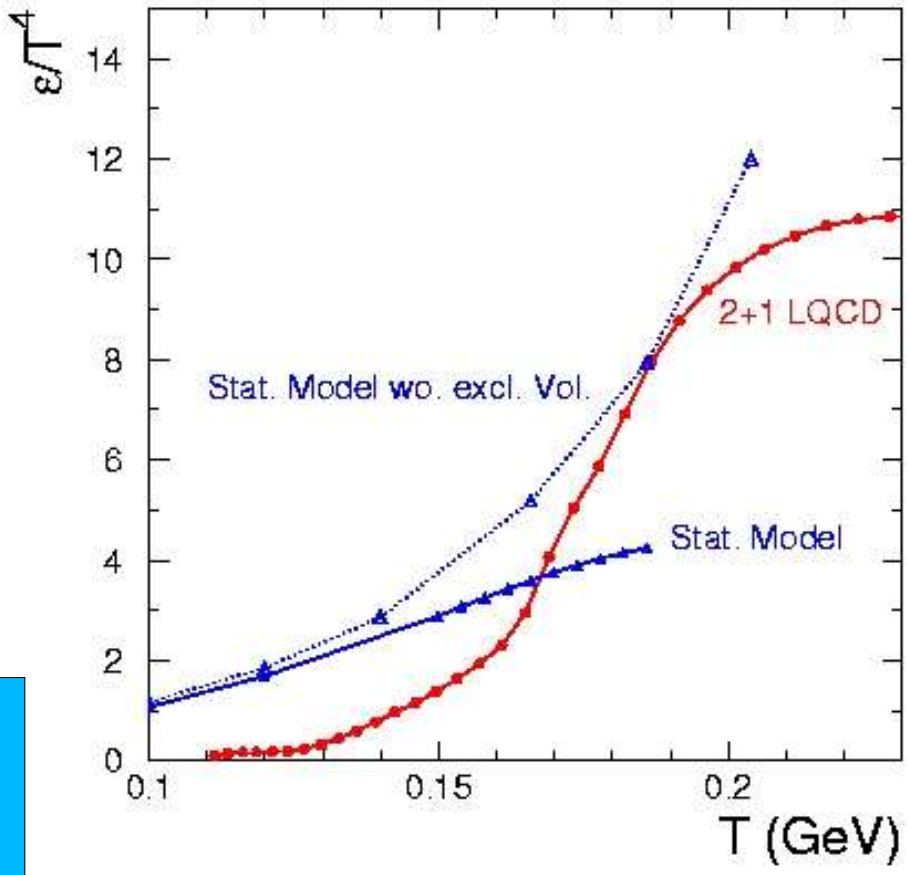


How is chemical equilibration achieved? Our Scenario

- Strangeness saturation takes place in the QGP phase.
- Phase transition is crossed from above.
- Near T_c new dynamics associated with collective excitations will take place and trigger the transition.
- Propagation and scattering of these collective excitations is expressed in the form of multi-hadron scattering. Near T_c multi-hadron processes will therefore be dominant. Chemical equilibrium is reached via these multi-hadron scattering events.

Chemical freeze-out takes place at T_c!

- Two-body collisions are not sufficient to bring multi-strange baryons into equilibrium.
- The density of particles varies rapidly with T near the phase transition.
- Multi-particle collisions are strongly enhanced at high density and lead to chem. eqilibrium very near to T_c.



pbm, J. Stachel, C. Wetterich
Phys. Lett. B596 (2004) 61
nucl-th/0311005

Lattice QCD calcs.
By F. Karsch et al.

Chemical Equilibration must take place in the Hadronic Phase

- Hadron yields determined by Boltzmann factors with 'free' vacuum masses.
- Particle distribution in QGP phase has no 'memory' of vacuum hadron masses .
- Relative yields are not determined by the strange quark mass but by individual strange hadron masses (at fixed T and m).
- But: the number of strange quarks is determined in the QGP phase! Equilibrium then implies redistribution of strange quarks.

Variation of fireball temperature with time

Values chosen appropriate for RHIC Au + Au collisions

- Assume: T_{ch}=176 MeV

  density decrease between chemical and thermal freeze-out: 30 %

- Two-pion correlation data: R_{side} =5.75 fm, R_{long}=7.0 fm, mean β_t =0.5, β_{long}=1

- Isentropic expansion → τ_f = 0.9 - 2.3 fm, T_f = 158 - 132 MeV

  (uncertainty due to variation in density profile)

- Near T_c: rate of decrease in temperature $|\dot{T}/T| = \tau_T^{-1} = (13 \pm 1)$ % /fm

2-body collisions are not enough

typical densities at T_{ch}: ρ_π =0.174/fm^3 (incl. res.) ρ_K =0.030/fm^3 ρ_Ω = 0.0003/fm^3

- To maintain equilibrium even for 5 MeV below T_{ch} need relative rate change

$$\left|\frac{\bar{r}_\Omega}{n_\Omega} - \frac{\bar{r}_K}{n_K}\right| = \tau_\Omega^{-1} - \tau_K^{-1} = (1.10 - 0.55)/\mathrm{fm} = 0.55/\mathrm{fm}.$$

So, Ω density needs to change by 100 % within 1 fm/c

- Typical reactions with large cross sections of 10 mb and relative velocity of 0.6 give

$\Omega + \pi \rightarrow \Xi + K$ $\rightarrow$ $\bar{r}_\Omega/n_\Omega = n_{\bar{\pi}}\langle v_r \sigma\rangle$=0.086/fm

$\pi + \pi \rightarrow K + \bar{K}$ (σ=3mb) $\rightarrow$ $\bar{r}_K/n_K = 0.18$/fm

i.e. much too slow to maintain equilibrium even over Δ T = 5 MeV!

- Even much more difficult: to produce large Ω abundancy

assume hadronization like in pp, factor 8 too few Ωs, to produce them within 1 fm/c

need reactions that provide $\bar{r}_\Omega/n_\Omega$=1.0 $\Rightarrow$ not with 2-body reactions

- Consensus in the literature: Koch, Müller, Rafelski, Phys. Rep. 142(1986), C. Greiner, S. Leupold, J.Phys.G27(2001)L95; P. Huovinen, J. Kapusta, nucl-th/0310051

Evaluation of multi-strange baryon yield

consider situation at T_{ch}=176 MeV first

- rate of change of density for n_{in} ingoing and n_{out} outgoing particles

$$r(n_{in}, n_{out}) = \bar{n}(T)^{n_{in}} |\mathcal{M}|^2 \phi$$

with

$$\phi = \prod_{k=1}^{n_{out}} \left(\int \frac{d^3 p_k}{(2\pi)^3 (2E_k)} \right) (2\pi)^4 \delta^4 \left(\sum_k p_k^\mu \right)$$

- The phase space factor ϕ depends on $\sqrt{s}$
  needs to be weighted by the probability $f(s)$ that multiparticle scattering occurs at a given value of $\sqrt{s}$
  evaluate numerically in Monte-Carlo using thermal momentum distribution
- typical reaction: $\Omega + \bar{N} \rightarrow 2\pi + 3K$
  assume cross section equal to measured value for $p + \bar{p} \rightarrow 5\pi$
  relevant $\sqrt{s} = 3.25$ GeV $\rightarrow \sigma$=6.4 mb
- compute matrix element and use for rate of $2\pi + 3K \rightarrow \Omega + \bar{N}$

Evaluation of multi-strange baryon yield

reaction $2\pi + 3K \rightarrow \Omega + \bar{N}$ leads to

$$r_\Omega = 0.00014\ \mathrm{fm}^{-4} \text{ or } r_\Omega / n_\Omega = 1/\tau_\Omega = 0.46/\mathrm{fm}$$

$\Rightarrow$ can achieve final density starting from 0 in 2.2 fm/c!

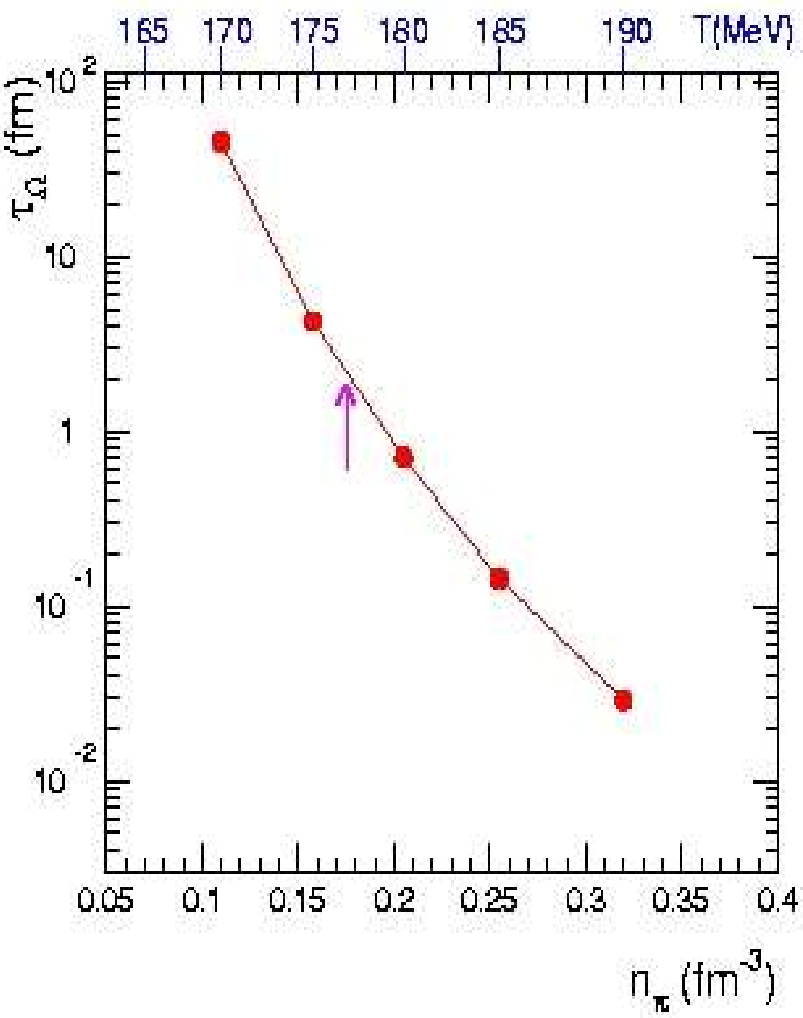
similarly one obtains

for $3\pi + 2K \rightarrow \Xi + \bar{N}$ $\quad \tau_\Xi = 0.71$ fm/c

and

for $4\pi + K \rightarrow \Lambda + \bar{N}$ $\quad \tau_\Lambda = 0.66$ fm/c

Density dependence of characteristic time for strange baryon production



- Near phase transition particle density varies rapidly with T.
- For small μ_b, reactions such as $KKK\pi\pi \rightarrow \Omega N_{bar}$ bring multi-strange baryons close to equilibrium.
- Equilibration time $\tau \propto T^{-60}$!
- All particles freeze out within a very narrow temperature window.

pbm, J. Stachel, C. Wetterich
Phys. Lett. B596 (2004)61
nucl-th/0311005

Check numerics via detailed balance

- Initially manifestly nonequilibrium situation - start with practially zero Ω density
- As equilibrium is approached
  rates $3K + 2\pi \rightarrow \Omega + \bar{N}$ and $\Omega + \bar{N} \rightarrow 3K + 2\pi$ have to become equal
- back and forth reactions scale very differently with pion density
  $\rightarrow$ only at one density can they be equal
- to explicitly check these rates now use pion, kaon, nucleon densities before strong decays, i.e. without resonance feeding
  (for all resonances corresponding rates have to be calculated accordingly)
- find: creation of Ω with $r_\Omega/n_\Omega = 3.4\ 10^{-3}/\mathrm{fm}$
  and annihilation of Ω with $r_\Omega/n_\Omega = 1.4\ 10^{-3}/\mathrm{fm}$

for equal rates reduce density by 25 %
reduce T by 2-3 MeV or excluded volume a bit larger

What about pp and e+e- collisions?

- Thermal fits describe hadron yields with T ~ 160 MeV
- Hadronization may be pre-thermalization process
- But: multi-strange baryons can only be reproduced by ad-hoc strangeness suppression factor implying incomplete equilibration

pp and e+e- continued

- Suppression factor of 2 implies Omega baryons are factor 8 off the equilibrium value
- Suppression is not due to canonical thermodynamics (phi problem, K. Redlich)
- Multi-meson fusion not effective since no high density phase
- 'Temperature' in pp and e+e- reflects hadronization but not phase transition.
- The existence of a medium in AA collisions also leads to the result that T is not universal (at T = 160 MeV as in e+e- and pp) but varies with μ: T=140 MeV at μ = 400 MeV, e.g.

Analysis of pp collisions

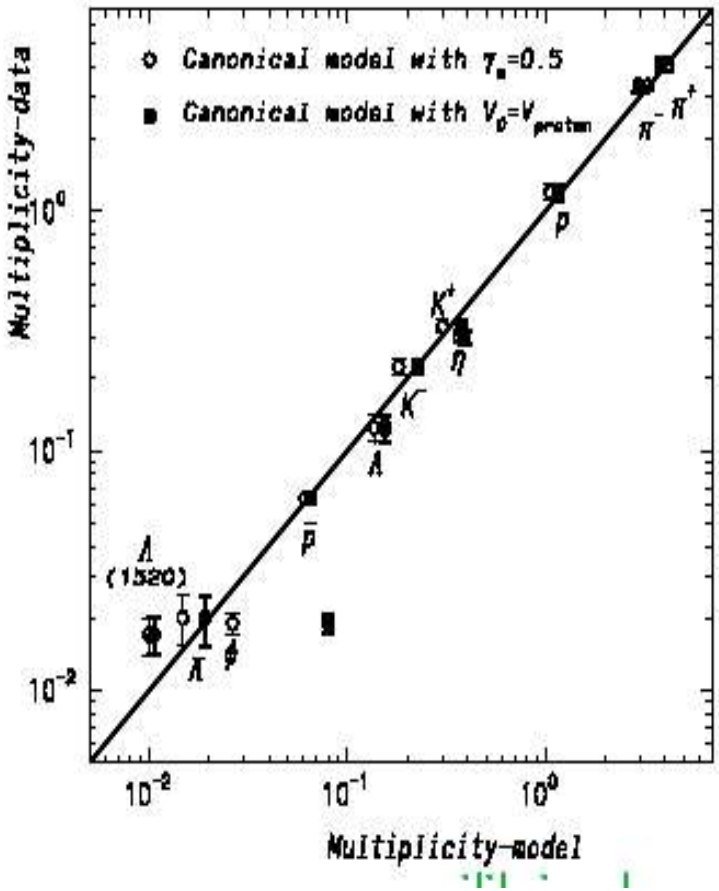
F. Becattini, Z. Phys. C69 (1996) 485; F. Becattini and U. Heinz, Z. Phys. C76 (1997) 269

pp data, $\sqrt{s} = 27.6$ GeV

canonical (volume) suppression vs γ_s factor (non-equilibrium), T = 165 MeV

Analysis by K. Redlich, see pbm, Stachel, Redlich, nucl-th/0304013

γ_s factor needed to describe ϕ production



Observed strangeness suppression is **not** described by equilibrium thermo-dynamics

What about centrality dependence of chemical equilibration?

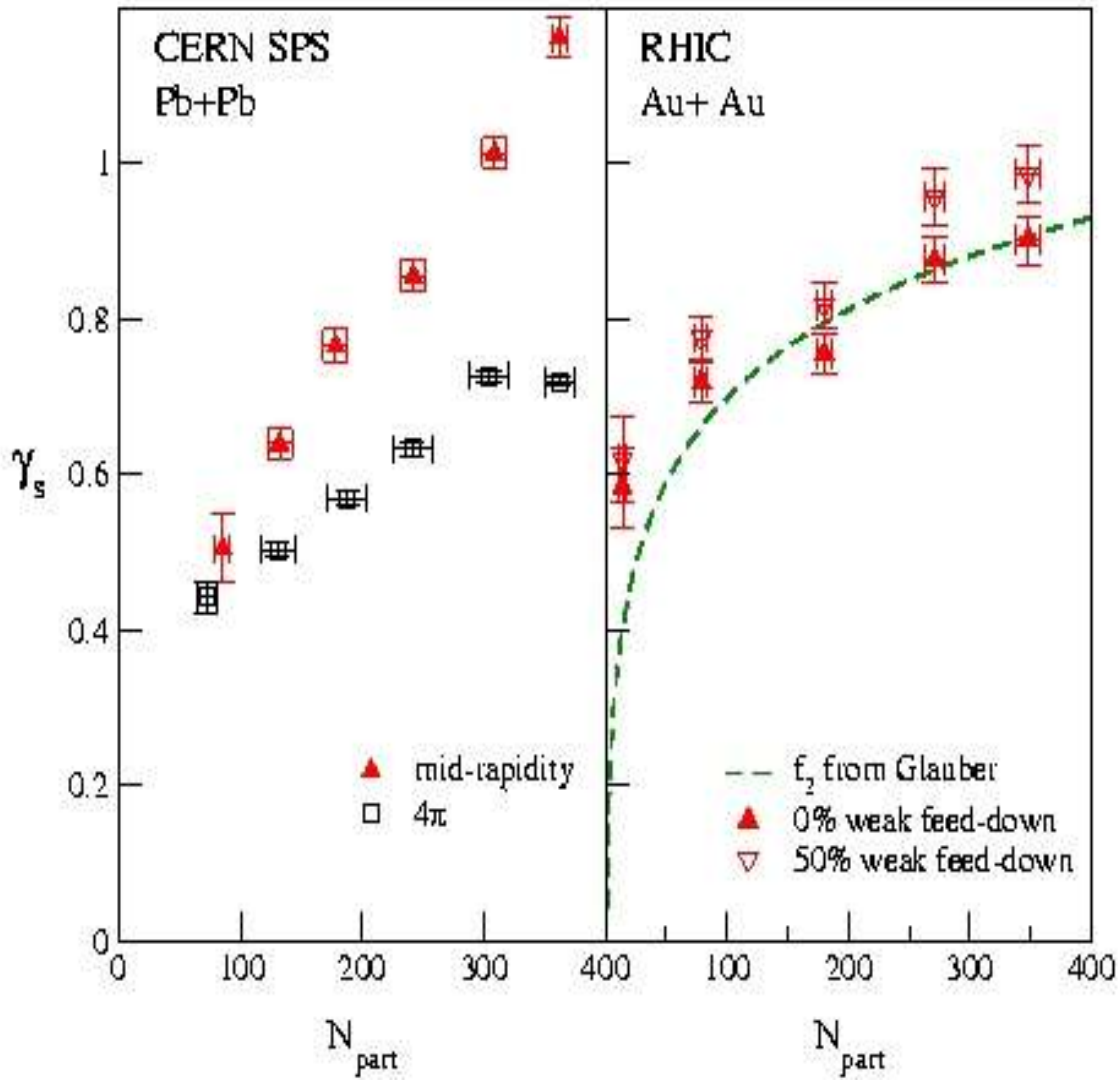
- Apparent chemical temperature depends little on centrality.
- The importance of multiple collisions should decrease with decreasing particle density, i.e. lower centrality.
- This is expressed in the data as change in γ_s .
- Note: $\gamma_s = 0.8$ reduces Ω yield by factor of 2.

Centrality dependence of γ_s

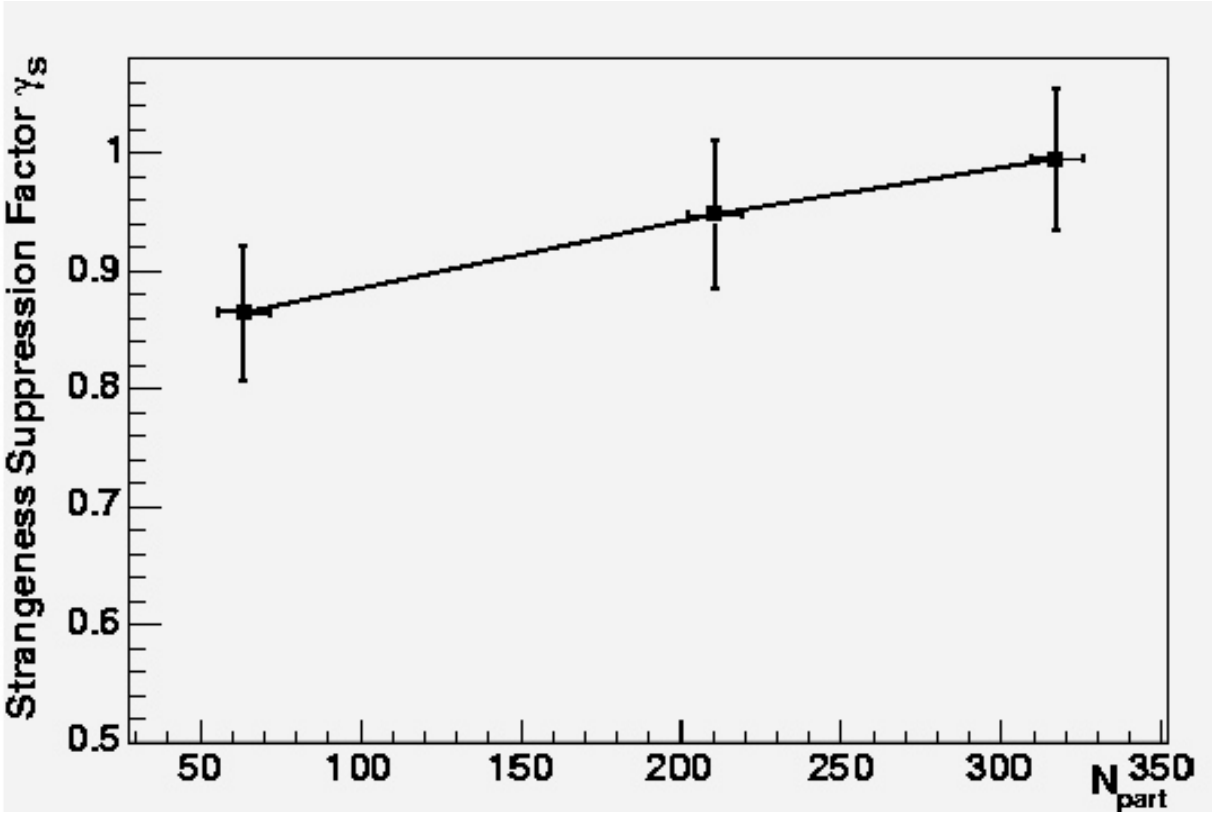
Cleymans, Kämpfer, Steinberg, Wheaton, hep-ph/0212335

Fit μ_B and γ_S to π, K, p yields

f_2 fraction of N_{part} with multiple collisions



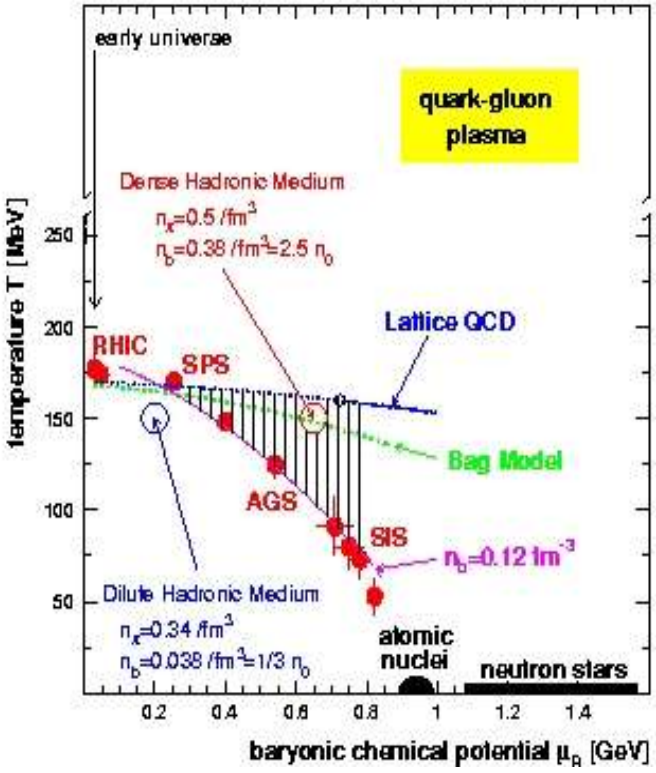
Centrality dependence of γ_s



S. Wheaton et al, SQM04,
Au+Au analysis, RHIC energy
increasing N_{part} → increasing particle density
→ chemical equilibration is reached

What about lower beam energies?

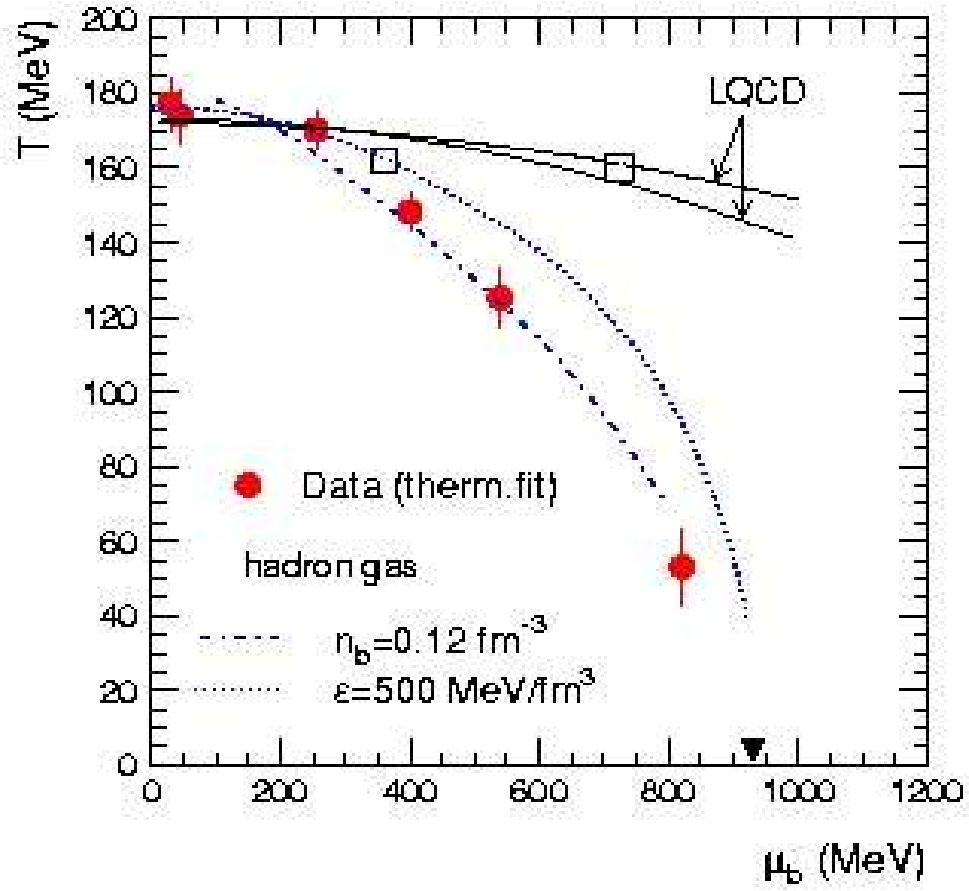
- at top SPS energy numbers work out nearly the same as at RHIC
- at 40 A GeV/c pion and kaon densities lower by 1/3 $\rightarrow \tau_\Omega$ increases by factor 12
- but: other reactions involving baryons must come into play at high baryon density: $N\rho KKK \rightarrow \Omega\pi$ or $N\pi\pi KKK \rightarrow \Omega\rho$



The QCD phase diagram and chemical freeze-out

Data are nearly described by curve of constant critical energy density

Conjecture: chemical freeze-out points delineate the QCD phase boundary also at larger μ down to AGS energy



A remark on critical energy density

- Along the Fodor-Katz phase boundary, critical energy density increases with increasing μ
- At $\mu = 0$, $\varepsilon_{crit} = 0.6$ GeV/fm^3
- At T = 160 MeV and μ = 650 MeV, $\varepsilon_{crit} \approx 2.7$ GeV/fm^3

  calc. within hadron resonance gas model, no excluded volume correction
- There are 1.46 baryons/fm^3 and 0.44 mesons/fm^3 at this point

This is certainly not a standard hadron resonance gas

Conclusions

- Chemical equilibration of multi-strange hadrons is obtained through multi-hadron collisions near (during) the phase transition.
- Chemical freeze-out at RHIC and top SPS energy coincides with phase boundary from LQCD.
- Experimental determination of critical temperature:
  $T_c = T_{chem}$ +8 -18 MeV
- Open questions:
  - Where is phase boundary at lower energies?
  - Is the full chemical freeze-out curve coincident with the phase boundary?
  - Are multi-strange hadrons also equilibrated at SIS energy?

Progress in determination of fundamental QCD parameters from nuclear collisions