# Chemical equilibrium study in nucleus-nucleus collisions at SPS 158A GeV

## Outline

Statistical hadronization model
Fit results
System size dependence of chemical equilibration
Two component model
Conclusions

# Statistical hadronization model

- Hadrons emitted from regions at statistical equilibrium
- A snapshot of the system at chemical freeze out (inelastic interactions cease)
- Ideal hadron-resonance gas, grand-canonical ensemble, mean primary particle multiplicities are given by:

$$\langle N_i \rangle = \frac{(2J_i+1)V}{(2\pi)^3} \int \frac{d^3p}{(\gamma_S^{ns}\lambda_i)^{-1} e^{\sqrt{p^2+m_i^2}/T} \pm 1} \qquad \lambda_i = e^{(\mu_B B_i + \mu_S S_i + \mu_Q Q_i)/T}$$

(provided that masses and charges are distributed among different clusters corresponding to a single large fireball)

- 6 parameters to be fitted: $T, V, \mu_B, \mu_S, \mu_Q, \gamma_S$
- Eliminate 2 by conditions: S==0 and $\frac{n_Q}{n_B} = \frac{Z}{A}$
- Total multiplicity of a hadron = primary production + contribution from the decay of heavier hadrons

# Small systems

- Small systems (p-p), exact conservation of charges "BSQ-canonical ensemble"
- Uses Boltzmann approximation, mean primary particle multiplicities reads:

$$\langle N_i \rangle = \frac{Z_{B-B_i, S-S_i, Q-Q_i}}{Z_{B,S,Q}} z_i^1$$

$$\omega = \sum_B z_i^1 e^{i(S_i \phi_S + Q_i \phi_Q)}$$

$$Z_{B,S,Q}(T, V, \gamma_S) = \frac{Z_0}{(2\pi)^2} \int_0^{2\pi} d\phi_S \int_0^{2\pi} d\phi_Q \cos(S\phi_S + Q\phi_Q - B \arg(\omega(\phi_S, \phi_Q)))$$

$$.\times I_B(2|\omega(\phi_S, \phi_Q)|) \exp(2\sum_M z_i^1 \cos(S_i \phi_S + Q_i \phi_Q))$$

- Exact conservation of Strangeness, "S-canonical ensemble"

$$\langle N_i \rangle = \frac{Z_{S-S_i}}{Z_S} \lambda_i z_i^1$$

$$\lambda_i = e^{(\mu_B B_i + \mu_Q Q_i)/T}$$

$$Z_{B,S,Q}(T, V, \gamma_S) = \frac{Z_0}{\pi} \int_0^{\pi} e^{\sum_i \gamma_S^{ns} \lambda_i z_i^1} [\cos(S\phi_S)\cos(\omega) + \sin(S\phi_S)\sin(\omega)] d\phi_S$$

$$\omega = \sum_i \gamma_S^{ns} \lambda_i z_i^1 \cos(S_i \phi_S)$$

# Fit results, NA49 p-p and Pb-Pb at 158AGeV

- No major discrepancy between data and model
- Theoretical multiplicities within 2 standard deviations

*From F. Becattini et al. Phys. Rev. C69: 024905, 2004*
*"Analysis B"*

| Parameter | Value | Error |
|---|---|---|
| T | 177.3 | 7.1 |
| $\gamma_s$ | 0.45 | 0.03 |
| V' | 0.13 | 0.01 |
| $\chi^2$/dof | 13/7 | |

$$V' = VT^3 \exp(-0.7/T)$$

BSQ- canonical ensemble

| Particle | Measurement | Fit | Residual |
|---|---|---|---|
| $\pi^+$ | 3.15±0.16 | 3.288 | 0.865 |
| $\pi^-$ | 2.45±0.12 | 2.455 | 0.042 |
| $K^+$ | 0.21±0.02 | 0.201 | -0.472 |
| $K^-$ | 0.13±0.013 | 0.108 | -1.828 |
| $K^0_S$ | 0.18±0.04 | 0.151 | -0.718 |
| $\phi$ | 0.012±0.0015 | 0.015 | 1.927 |
| $\bar{p}$ | 0.040±0.007 | 0.044 | 0.638 |
| $\Lambda$ | 0.115±0.012 | 0.117 | 0.179 |
| $\bar{\Lambda}$ | 0.0148±0.0019 | 0.014 | -0.448 |
| $\Lambda(1520)$ | 0.012±0.003 | 0.006 | -1.964 |

**NA49 p-p 158A GeV**

# Fit results, NA49 p-p and Pb-Pb at 158AGeV

- No major discrepancy between data and model
- Theoretical multiplicities within 2.5 standard deviations

*From F. Becattini et al. Phys. Rev. C69: 024905, 2004*
*"Analysis B"*

| Parameter | Value | Error |
|---|---|---|
| T | 154.8 | 2.1 |
| $\mu_B$ | 244.5 | 7.8 |
| $\gamma_S$ | 0.94 | 0.04 |
| V' | 18.46 | 1.07 |
| $\chi^2$ /dof | 21.6/9 | |

Grand canonical ensemble

| Particle | Measurement | Fit | Residual |
|---|---|---|---|
| $N_P$ | $362 \pm 1 \pm 5$ | 364 | 0.337 |
| $\pi^+$ | $619 \pm 17 \pm 31$ | 533 | -2.427 |
| $\pi^-$ | $639 \pm 17 \pm 31$ | 565 | -2.077 |
| $K^+$ | $103 \pm 5 \pm 5$ | 104 | 0.129 |
| $K^-$ | $51.9 \pm 1.9 \pm 3$ | 59.4 | 2.099 |
| $K_S^0$ | $81 \pm 4$ | 83 | 0.533 |
| $\phi$ | $7.6 \pm 1.1$ | 8.7 | 0.978 |
| $\Lambda$ | $53 \pm 5.0$ | 56 | 0.645 |
| $\bar{\Lambda}$ | $4.64 \pm 0.32$ | 5.08 | 1.366 |
| $\Xi^-$ | $4.45 \pm 0.22$ | 4.34 | -0.524 |
| $\bar{\Xi}^+$ | $0.83 \pm 0.04$ | 0.79 | -0.922 |
| $\Omega$ | $0.62 \pm 0.09$ | 0.49 | -1.438 |
| $\bar{\Omega}$ | $0.20 \pm 0.03$ | 0.19 | -0.388 |

**NA49 Pb-Pb 158A GeV**
(5% most central)

$$V' = VT^3 \exp(-0.7/T)$$

# Fit results, NA49 C-C

S- canonical ensemble

- No major discrepancy between data and model
- Theoretical multiplicities within a standard deviation

| Particle | Measurement | Fit | Residual |
|---|---|---|---|
| $N_P$ | $16.3 \pm 1$ | 16.0 | -0.339 |
| $\pi^+$ | $22.4 \pm 0.3 \pm 1.6$ | 22.0 | -0.229 |
| $\pi^-$ | $22.2 \pm 0.3 \pm 1.6$ | 22.0 | -0.093 |
| $K^+$ | $2.54 \pm 0.03 \pm 0.25$ | 2.75 | 0.829 |
| $K^-$ | $1.49 \pm 0.05 \pm 0.15$ | 1.51 | 0.104 |
| $\phi$ | $0.18 \pm 0.01 \pm 0.02$ | 0.16 | -1.103 |
| $\Lambda$ | $1.32 \pm 0.05 \pm 0.32$ | 1.69 | 1.152 |
| $\bar{\Lambda}$ | $0.181 \pm 0.006 \pm 0.019$ | 0.182 | 0.046 |

PRELIMINARY

Nucl-ex/0406031

**NA49 C-C 158A GeV**

(15.3% most central)

| Parameter | Value | Error |
|---|---|---|
| T | 165.7 | 4.1 |
| $\mu_B$ | 248.1 | 12.5 |
| $\gamma_S$ | 0.58 | 0.04 |
| V' | 0.84 | 0.05 |
| $\chi^2/\text{dof}$ | 3.4/4 | |

$$V' = VT^3 \exp(-0.7/T)$$

# Fit results, NA49 Si-Si

S- canonical ensemble

- No major discrepancy between data and model
- Theoretical multiplicities within 2 standard deviations

| Particle | Measurement | Fit | Residual |
|---|---|---|---|
| $N_P$ | $41.4 \pm 2$ | 40.3 | -0.553 |
| $\pi^+$ | $56.6 \pm 0.7 \pm 4.0$ | 56.5 | -0.032 |
| $\pi^-$ | $57.6 \pm 0.6 \pm 4.0$ | 56.5 | -0.265 |
| $K^+$ | $7.44 \pm 0.08 \pm 0.74$ | 8.17 | 0.985 |
| $K^-$ | $4.42 \pm 0.04 \pm 0.44$ | 4.49 | 0.148 |
| $\phi$ | $0.66 \pm 0.03 \pm 0.08$ | 0.51 | -1.841 |
| $\Lambda$ | $3.88 \pm 0.16 \pm 0.56$ | 4.760 | 1.697 |
| $\bar{\Lambda}$ | $0.505 \pm 0.015 \pm 0.076$ | 0.508 | 0.040 |

PRELIMINARY

Nucl-ex/0406031

**NA49 Si-Si 158A GeV**

(12.2% most central)

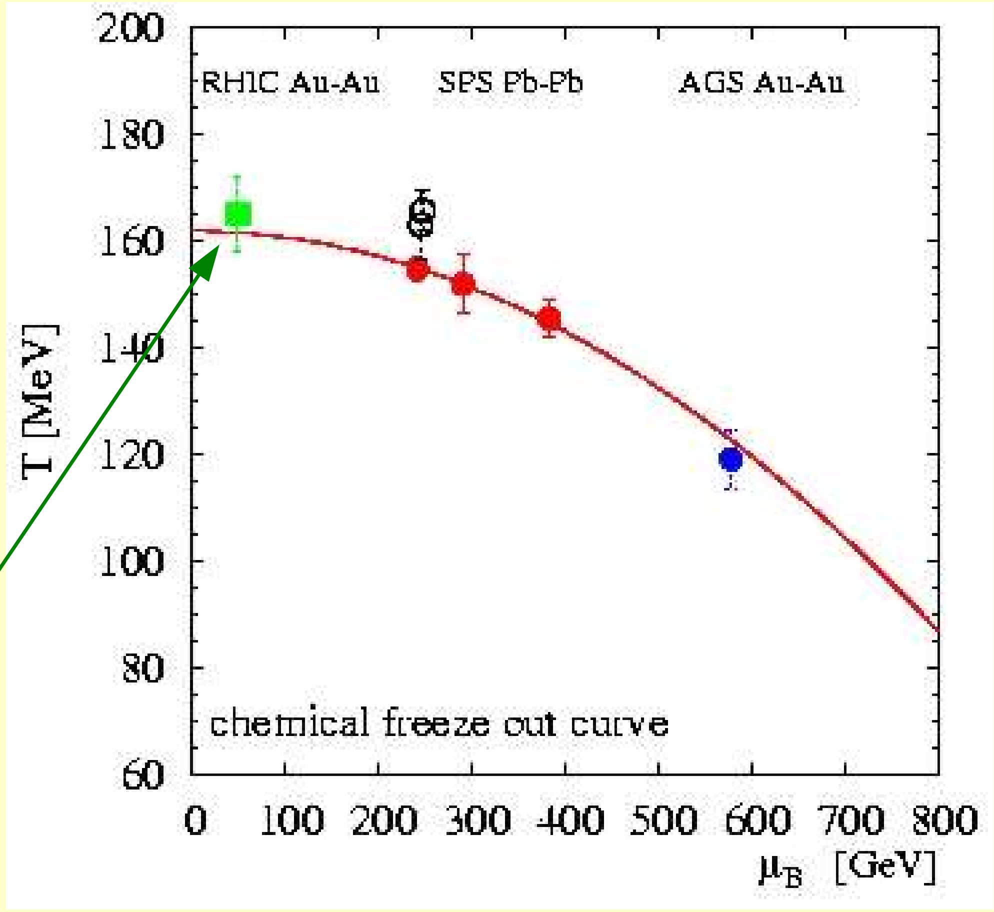
| Parameter | Value | Error |
|---|---|---|
| T | 163 | 6.5 |
| $\mu_B$ | 245.5 | 15.1 |
| $\gamma_S$ | 0.66 | 0.07 |
| V' | 2.1 | 0.18 |
| $\chi^2/\text{dof}$ | 7.6/4 | |

Errors re-scaled with $\sqrt{\chi^2/dof}$

$$V' = VT^3 \exp(-0.7/T)$$

# Chemical freeze-out curve

- Fitted parameters slightly above heavy system chemical FO curve
- At the same beam energy, baryon chemical potential weakly dependent on Np $\mu_B \approx 250\ \text{MeV}$
- At same beam energy, temperature 5-15 MeV larger than in heavy systems
- Small systems decouple at higher $\langle E \rangle / \langle N \rangle$ than heavy systems

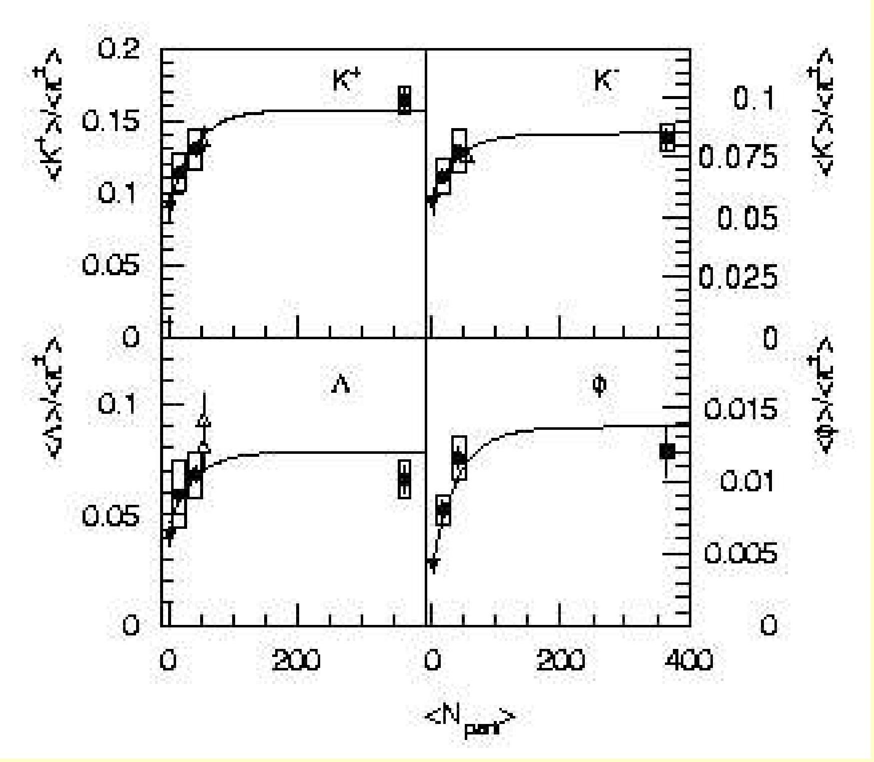


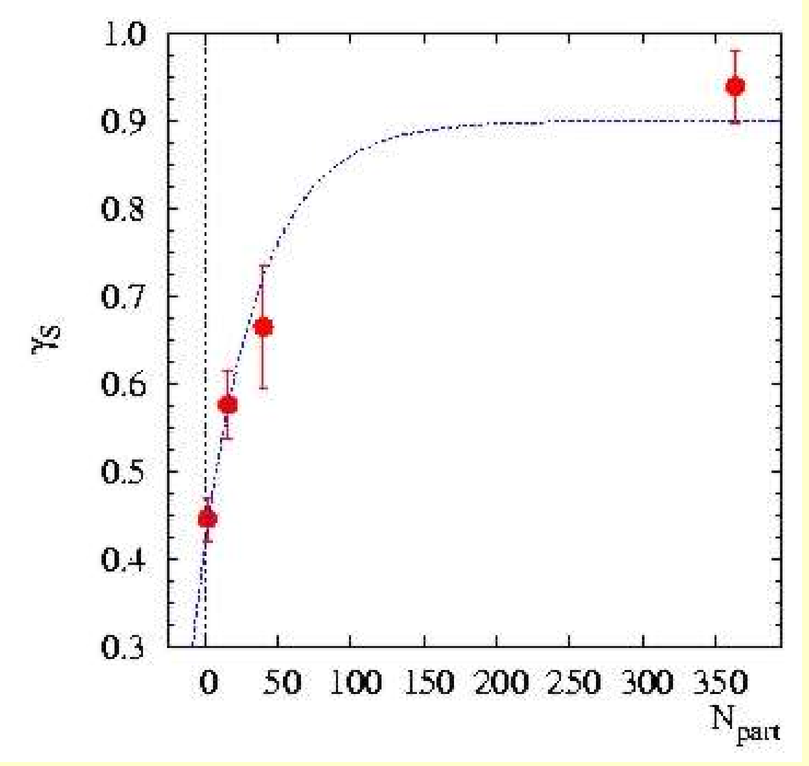

*From W. Florkowski et al.*
*Acta Phys. Polon. B33 (2002) p761*

# System size dependence: $\gamma_S$

*Taken from nucl-ex/0406031*

$\gamma_S$ rapidly increasing in $N_P$



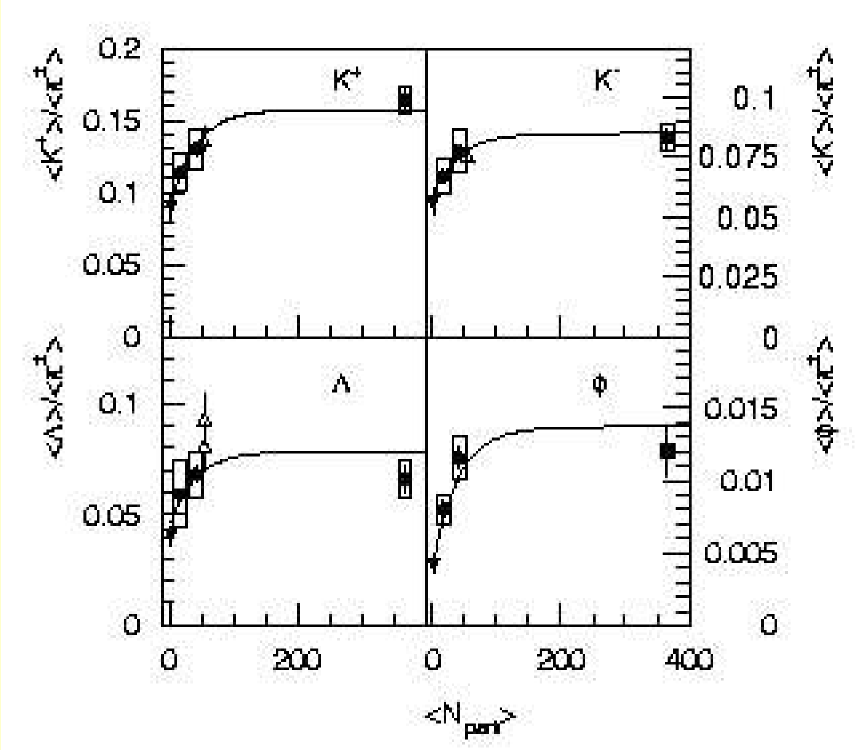

Curves to guide eye, functional form: a-b exp(-Np/40)

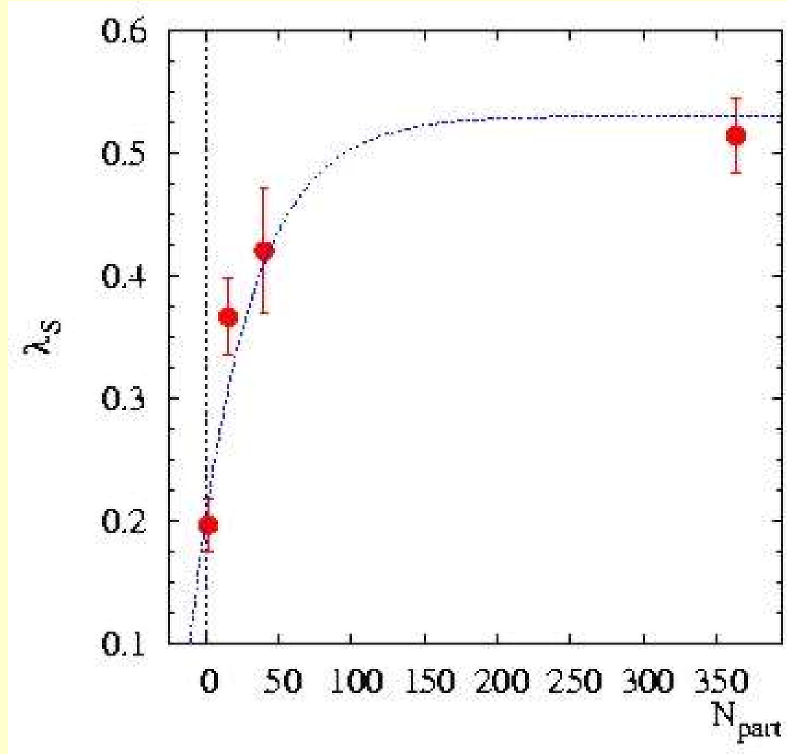
# System size dependence: $\lambda_s$

Wrobleski factor, similar behaviour

$$\lambda_s = \frac{2(\text{newly produced } s\bar{s})}{\text{newly produced } (u\bar{u} + d\bar{d})}$$

*Taken from nucl-ex/0406031*

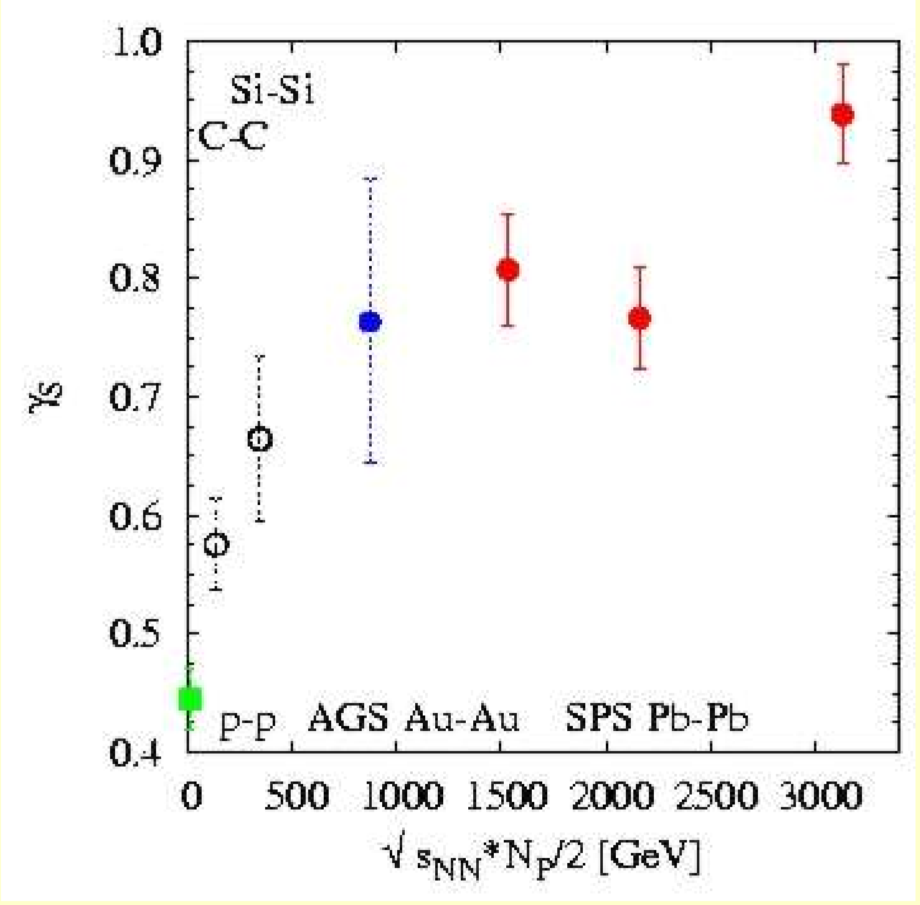


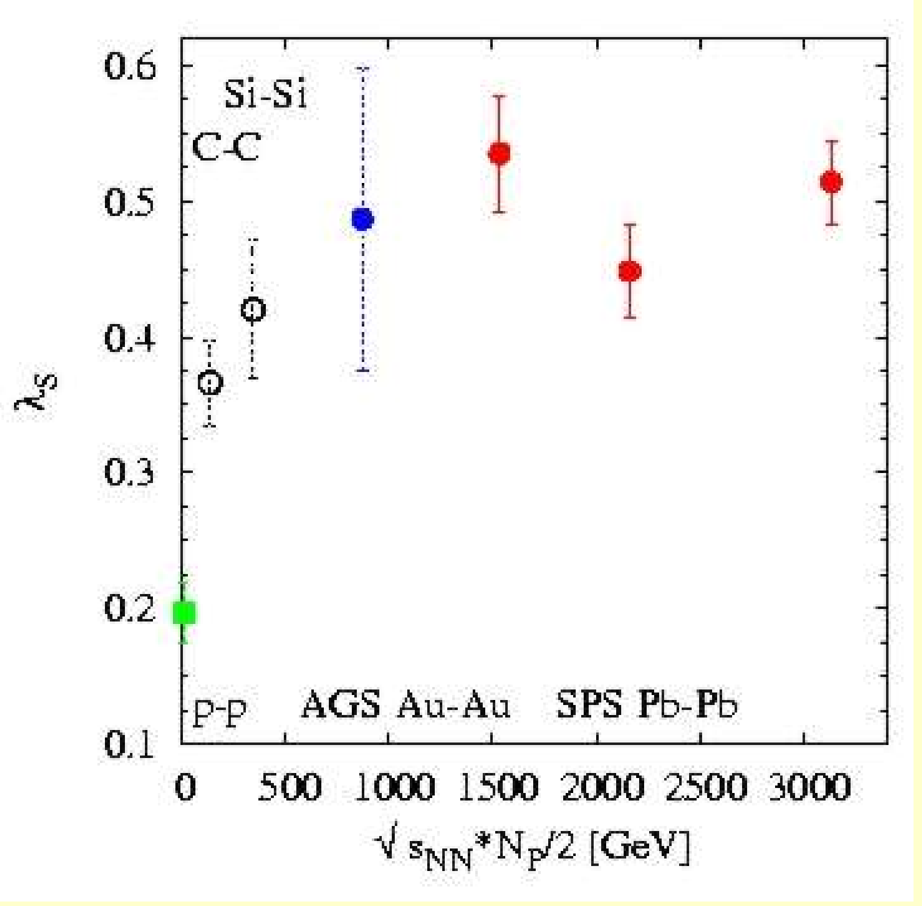




Curves to guide eye, functional form: a-b exp(-Np/40)

# Total energy dependence

**Smooth (monotonic) behaviour in strangeness production**



# Superposition of NN collisions with fully equilibrated fireball

- Divide system into two components:

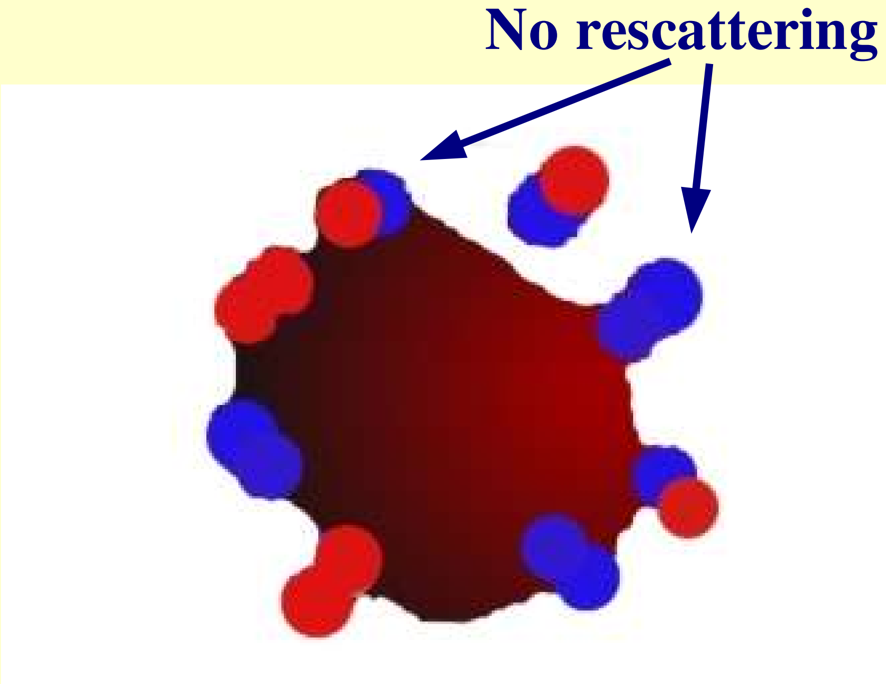
  Large cluster in complete chemical equilibrium $(\gamma_S = 1)$

  Strangeness suppression arising from single nucleon-nucleon collisions

- Fit pp yields at same energy
- (pp at 158A GeV, NA49) using BSQ-canonical ensemble
- Retain parameters from pp fit, calculate production from nn and np

$$N_i = N_i^{fireball} + \langle N_c \rangle N_i^{NN}$$

- Fit Nc together with the thermal parameters of equilibrated fireball



# Superposition of NN collisions with fully equilibrated fireball

- Good agreement with the standard statistical model
- 25 independent NN collisions ⇒ 310 nucleons out of 360 contribute to the formation of large equilibrated fireball
- 7 % $\pi$, 13 % proton, 0.004 % $\Omega$ coming from NN component
- ⟨Nc⟩ expected to be roughly the same for 20, 30, 40 and 80 A GeV collisions

NA49 Pb-Pb 158A GeV

| Parameters | SHM | SHM(TC) |
|---|---|---|
| Pb-Pb 158A GeV | | |
| $T$ (MeV) | 154.8±1.4 (2.1) | 153.9±1.3 (2.0) |
| $\mu_B$ (MeV) | 244.5±5.0 (7.8) | 239.4±6.3 (9.2) |
| $\gamma_S$ | 0.938±0.027 (0.042) | 1.0 (fixed) |
| $VT^3 \exp[-0.7\,\mathrm{GeV}/T]$ | 18.46±0.69 (1.07) | 16.26±0.56 (0.81) |
| $\langle N_c \rangle$ | | 25±9 (13) |
| $\chi^2$/dof | 21.6/9 | 19.1/9 |

# Superposition of NN collisions with fully equilibrated fireball

- Good agreement with the standard statistical model
- Significant improvement in fit quality
- 11 independent NN collisions ⇒ only half of the nucleons contribute to the formation of large equilibrated fireball
- 54 % $\pi$,  14 % proton,  0.08 % $\Omega$ coming from NN component

NA49 Si-Si 158A GeV

| Parameters | SHM | SHM(TC) |
|---|---|---|
| Si-Si 158A GeV | | |
| $T$ (MeV) | 163.0±4.7 (6.5) | 162.0±7.6 (7.6) |
| $\mu_B$ (MeV) | 245.5±11.0 (15.1) | 234.4±22.5 (22.5) |
| $\gamma_S$ | 0.664±0.050 (0.069) | 1.0 (fixed) |
| $VT^3 \exp[-0.7\,\mathrm{GeV}/T]$ | 2.10±0.13 (0.18) | 0.91±0.11 (0.11) |
| $\langle N_c \rangle$ | | 11.4±1.8 (1.8) |
| $\chi^2$/dof | 7.6/4 | 1.0/4 |

# Superposition of NN collisions with fully equilibrated fireball

- 6 independent NN collisions ⇒ 4 nucleons out of 16 contribute to the formation of large fireball
- Fire ball treated S-canonically, repeat analysis in BSQ-canonical ensemble

NA49 C-C 158A GeV

| Parameters | SHM | SHM(TC) |
|---|---|---|
| C-C 158A GeV | | |
| $T$ (MeV) | 165.7±4.1 (4.1) | 169.1±11.6 (13.3) |
| $\mu_B$ (MeV) | 248.1±12.5 (12.5) | 238.3±43.7 (50.0) |
| $\gamma_S$ | 0.575±0.042 (0.042) | 1.0 (fixed) |
| $VT^3\exp[-0.7\,\mathrm{GeV}/T]$ | 0.84±0.05 (0.05) | 0.23±0.04 (0.04) |
| $\langle N_c \rangle$ | | 5.7±0.7 (0.8) |
| $\chi^2$/dof | 3.4/4 | 5.2/4 |

# Conclusions

- Statistical model fits well all the NA49 data analysed here
- T and $\mu_B$, weak Np dependence. $\gamma_S$ stronger Np dependence
- T and $\mu_B$, strong energy dependence. $\gamma_S$ weaker energy dependence
- Strangeness under saturation might be possible to explain without $\gamma_S$ with the two component hadronization model