Quasi-particle model for deconfined matter

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- Comparison with lattice data
- Expansion coefficients $c_i(T)$ in pressure correction
- Contact of QPM with QCD

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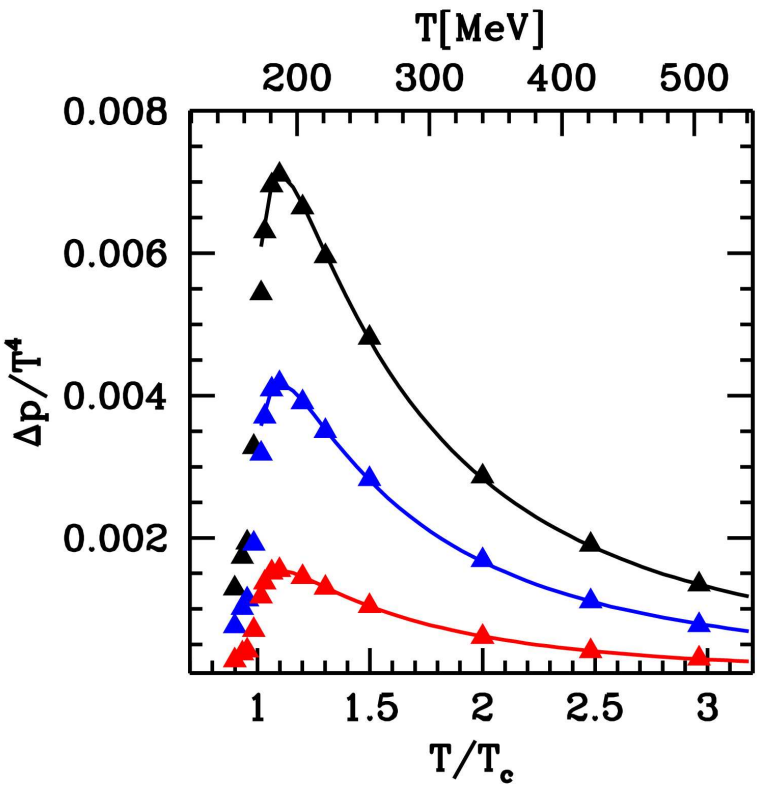
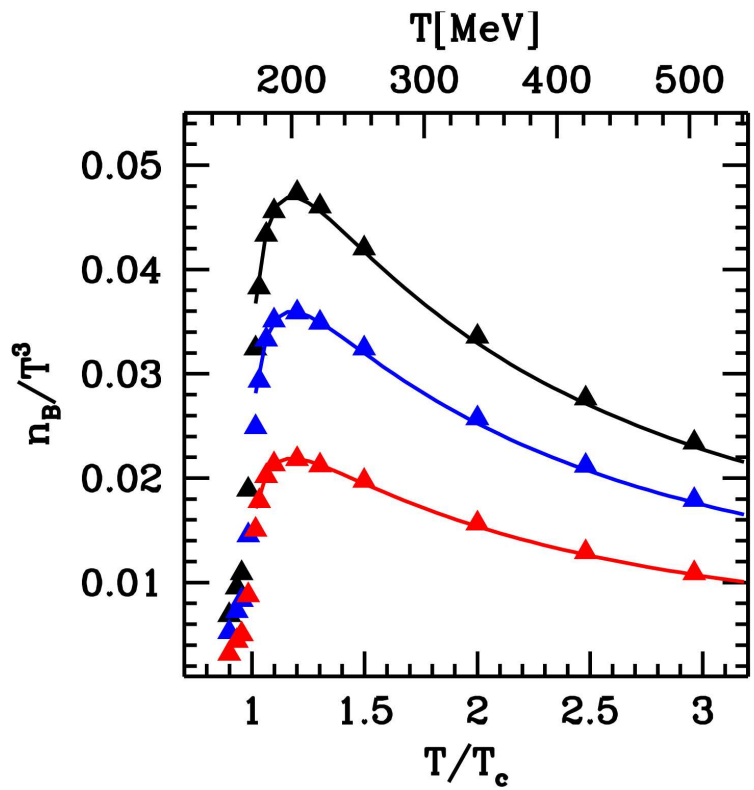
Comparison of QPM with lattice data

$$\Delta p(T,\mu) = p(T,\mu) - p(T,\mu=0)$$

$$\frac{n_B}{T^3} = T\,\frac{\partial}{\partial \mu_B}\Big|_T \left(\frac{\Delta p}{T^4}\right)$$

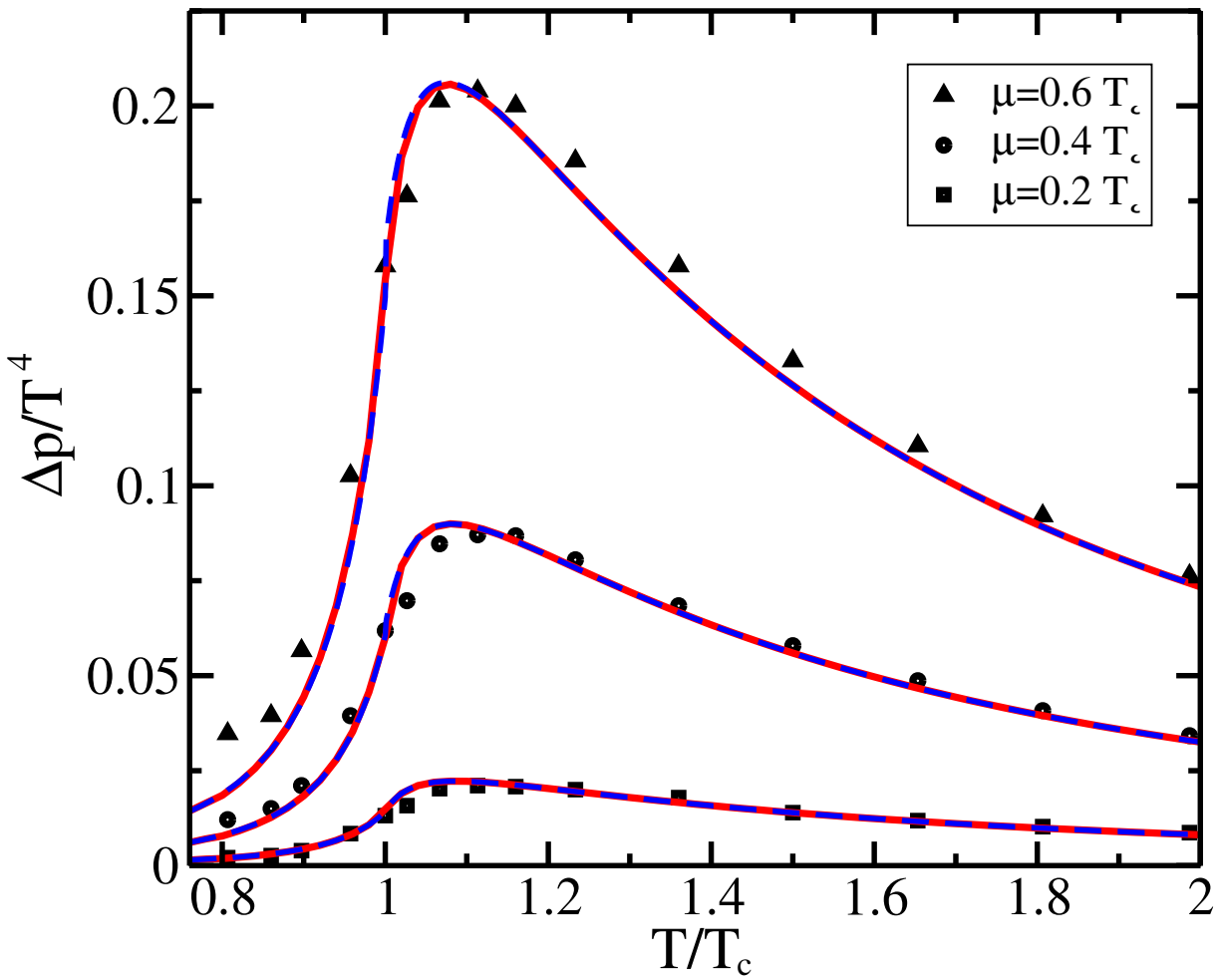




Fodor et al. 2002
Szabo et al. 2003
$N_f = 2+1$

Pressure correction for $\mu > 0$

$$\frac{\Delta p(T,\mu)}{T^4} = c_2(T)\left(\frac{\mu}{T}\right)^2 + c_4(T)\left(\frac{\mu}{T}\right)^4$$

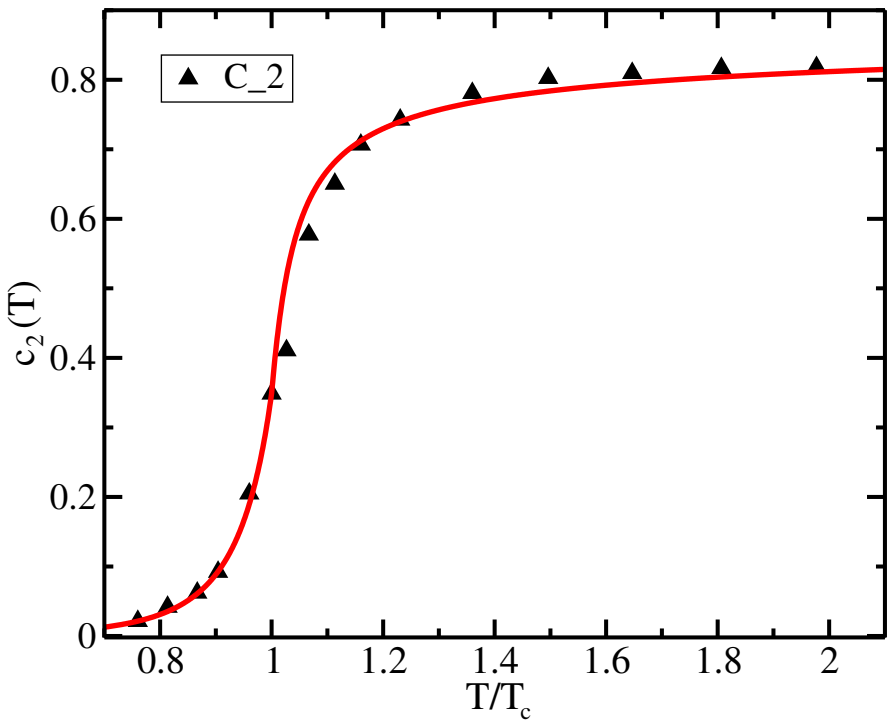


Allton et al. 2003
$N_f = 2$

Expansion coefficients $c_i(T)$

$$\frac{p(T,\mu)}{T^4} = c_0(T) + c_2(T)\left(\frac{\mu}{T}\right)^2 + c_4(T)\left(\frac{\mu}{T}\right)^4 + \ \ldots$$

$$c_n(T) = \frac{1}{n!}\frac{\partial^n p}{\partial \mu^n}\bigg|_{\mu=0}$$



$c_0(T)$ (cf. B. Kämpfer)

$$c_2 \propto \int dk \ \ldots \ G^2\Big|_{\mu=0}$$

$$\omega_i^2 = k^2 + m_i^2 \sim G^2$$

Allton et al. 2003

$N_f = 2$

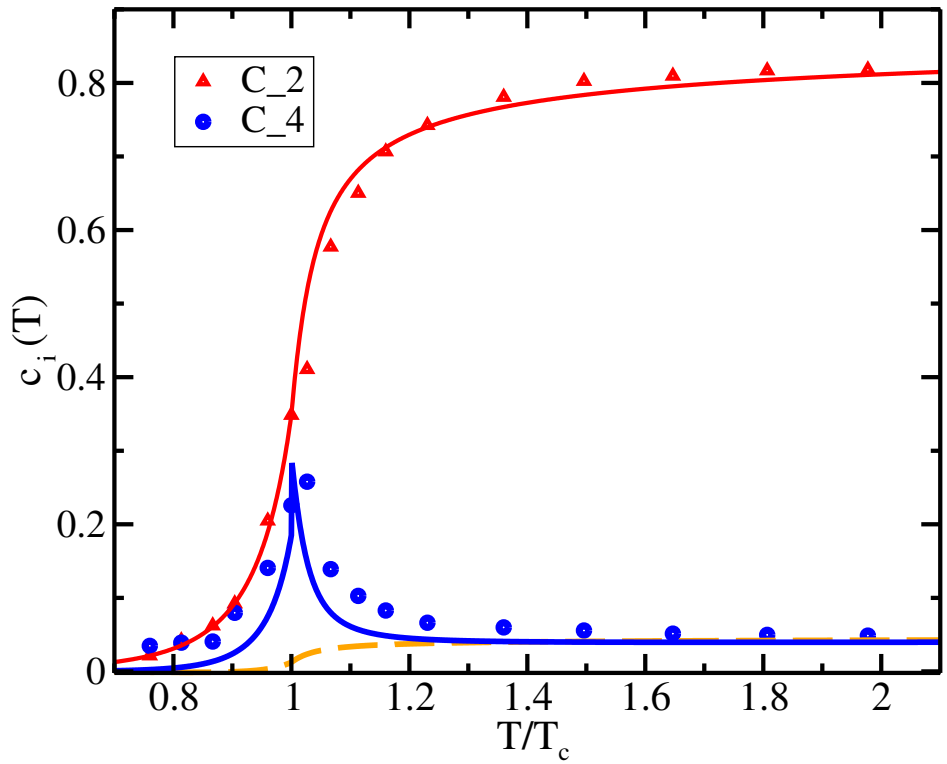
Expansion coefficients $c_i(T)$ - cont'd

$$c_4 \propto \int dk \left\{ \ldots G^2 + \ldots \frac{\partial^2 G^2}{\partial \mu^2} \right\}\Bigg|_{\mu=0}$$

$G^2(T) \Longleftrightarrow c_2$
consistency check for c_4

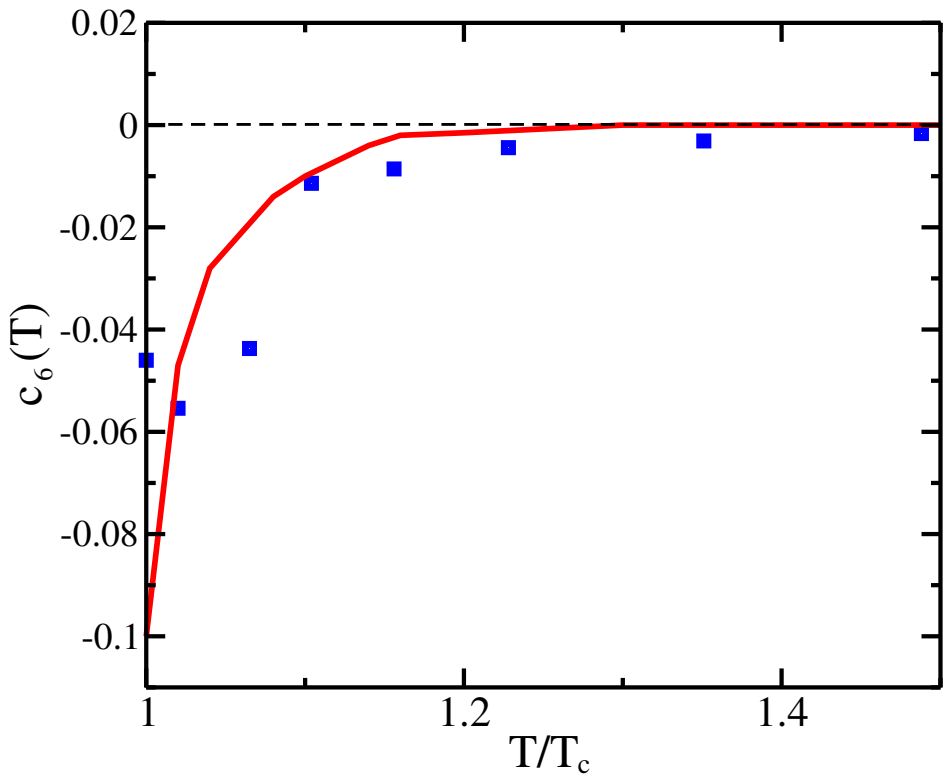
peak in c_4 caused by
$\frac{\partial^2 G^2}{\partial \mu^2}\Big|_{\mu=0}$

Allton et al. 2003
$N_f = 2$



Expansion coefficient $c_6(T)$

$$c_6 \propto \int dk \left\{ \ldots G^2 + \ldots \frac{\partial^2 G^2}{\partial \mu^2} + \ldots \frac{\partial^4 G^2}{\partial \mu^4} \right\}\Bigg|_{\mu=0}$$



$\Longrightarrow$ $\left.\frac{\partial^4 G^2}{\partial \mu^4}\right|_{\mu=0}$ more involved

under consideration

Quasi-particle model

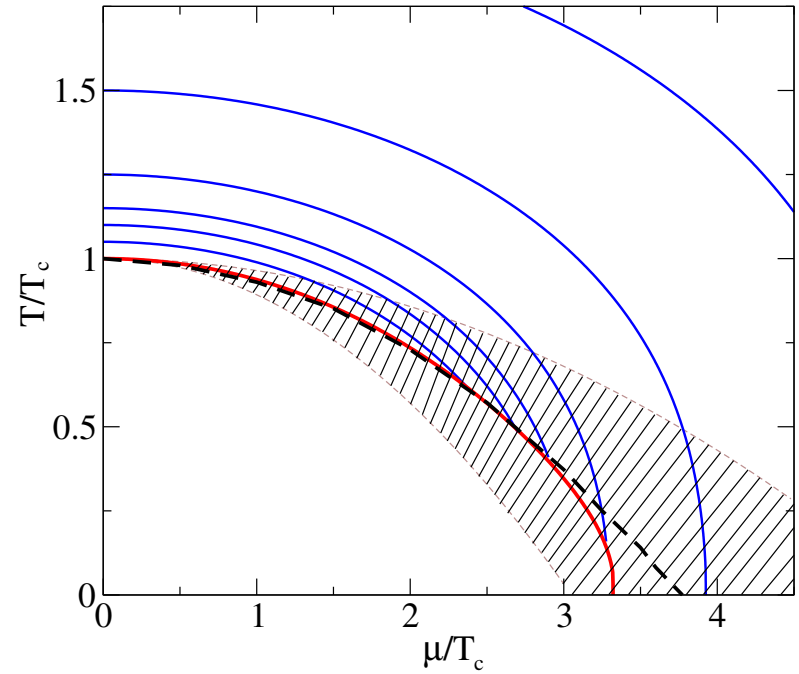
$$a_T \frac{\partial G^2}{\partial T} + a_\mu \frac{\partial G^2}{\partial \mu} = b$$

Cauchy: $G^2(T, \mu = 0)$

curvature: $$T_c \left. \frac{\partial^2 T_c}{\partial \mu^2} \right|_{\mu=0} = -0.122$$

(QPM)

$\rightarrow -0.14(6)$ (lattice)



– – – Allton et al. 2002

$$s(T,\mu) = \sum_{i=q,s,g} s_i(T,\mu)$$

$$s_i(T,\mu) = \frac{d_i}{2\pi^2 T} \int_0^\infty dk k^2 \left\{ \frac{\frac{4}{3}k^2 + m_i^2}{\sqrt{k^2 + m_i^2}} \left[f_+(\omega_i) + f_-(\omega_i) \right] - \mu_i \left[f_+(\omega_i) - f_-(\omega_i) \right] \right\}$$

Φ-derivable approximation scheme

problem: expansion of $\Omega(g) \Longleftrightarrow g \gtrsim 1$

- HTL-resummation $\Longrightarrow$ improvement
  J. O. Andersen, E. Braaten and M Strickland (1999,2000)
- Φ-derivable approximation scheme $\Longrightarrow$ $T \gtrsim 2.5 T_c$
  J. P. Blaizot, E. Iancu and A. Rebhan (2001)

$$s = -\left.\frac{\partial p}{\partial T}\right|_{\mu} = -\left.\frac{\partial(\Omega/V)}{\partial T}\right|_{\mu}$$

$$\Omega[D,S] = T\left(\frac{1}{2}\mathrm{Tr}\left[\ln D^{-1} - \Pi D\right] - \mathrm{Tr}\left[\ln S^{-1} - \Sigma S\right]\right) + T\Phi[D,S]$$

Dyson's equations: $\Pi[D] = D^{-1} - D_0^{-1}$; $\Sigma[S] = S^{-1} - S_0^{-1}$

stationarity property of Ω: $\frac{\delta\Omega[D,S]}{\delta D} = 0 \quad \Longrightarrow \quad \frac{\delta\Phi[D,S]}{\delta D} = \frac{1}{2}\Pi$

$\frac{\delta\Omega[D,S]}{\delta S} = 0 \quad \Longrightarrow \quad \frac{\delta\Phi[D,S]}{\delta S} = \Sigma$

Φ-derivable approximation scheme - cont'd

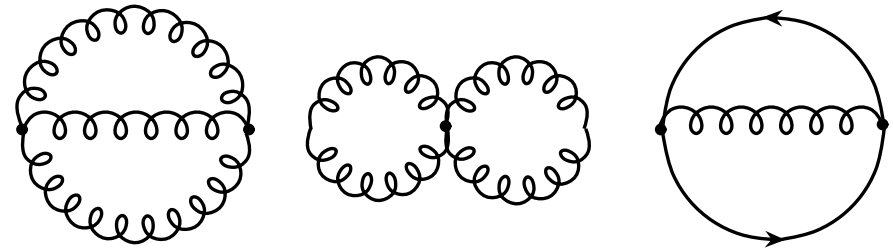
- perform trace Tr $\Longrightarrow$ sum over Matsubara frequencies ω_n
- remaining trace tr $\Longrightarrow$ colour, flavour
- retarded propagators

$$s = s_g + s_q + s'$$

$$\begin{aligned}
s_g &= -\mathrm{tr} \int \frac{d^4k}{(2\pi)^4} \frac{\partial n(\omega)}{\partial T} \left(\mathrm{Im} \left[\ln D^{-1} \right] - \mathrm{Im}\,\Pi\, \mathrm{Re} D \right) \\
s_q &= -2\,\mathrm{tr} \int \frac{d^4k}{(2\pi)^4} \frac{\partial f(\omega)}{\partial T} \left(\mathrm{Im} \left[\ln S^{-1} \right] - \mathrm{Im}\,\Sigma\, \mathrm{Re} S \right) \\
s' &= - \left. \frac{\partial (T\Phi[D,S]/V)}{\partial T} \right|_{D,S} \\
&\quad + \mathrm{tr} \int \frac{d^4k}{(2\pi)^4} \frac{\partial n(\omega)}{\partial T} \mathrm{Re}\Pi\, \mathrm{Im} D + 2\,\mathrm{tr} \int \frac{d^4k}{(2\pi)^4} \frac{\partial f(\omega)}{\partial T} \mathrm{Re}\Sigma\, \mathrm{Im} S
\end{aligned}$$

Φ-derivable approximation scheme - cont'd

- truncate Φ at 2-loop order
- modified Π, Σ
- $s' = 0$



- $D_L, D_T \quad \rightarrow \quad$ neglect D_L
  $S_+, S_- \quad \rightarrow \quad$ neglect S_- $\quad \Longrightarrow$ only dominating D_T, S_+ modes
- gauge invariance ? $\rightarrow$ approximately self-consistent:
  gauge invariant contributions

$\Longrightarrow$ employ HTL-expressions: $\quad \hat{\Pi}_T, \hat{\Sigma}_+ \longrightarrow \hat{D}_T, \hat{S}_+$

$$s^{HTL} = s_g^{HTL} + s_q^{HTL}$$

Φ-derivable approximation scheme - cont'd

- rewrite $\mathrm{Im}(\ln \hat{D}_T^{-1})$, $\mathrm{Im}(\ln \hat{S}_+^{-1})$
  $\Longrightarrow$ neglect Landau damping

$$
\begin{aligned}
s_{g,QP}^{HTL} &= 2(N_c^2-1)\int \frac{d^3k}{(2\pi)^3}\sigma(\hat{\omega}_{T,k}) \\
\sigma(\hat{\omega}_{T,k}) &= -\ln(1-e^{-\beta\hat{\omega}_{T,k}}) + \beta\hat{\omega}_{T,k}n(\hat{\omega}_{T,k}) \\
\hat{\omega}_{T,k} &\longleftarrow \omega_T^2 - k^2 - \hat{\Pi}_T(\omega_T,k) = 0
\end{aligned}
$$

- neglect momentum- and energy-dependence in $\hat{\Pi}_T$, $\hat{\Sigma}_+$
  $\hat{\omega}_{T,k} \to \omega_g = \sqrt{k^2+m_\infty^2} \quad \Leftarrow \quad m_\infty^2 = \frac{1}{2}\hat{m}_D^2$
  $\Longrightarrow$ QPM-expression of $s = s(T,\mu)$
- p from s up to integration constant

Conclusion

- QPM describes lattice data: $N_f = 2 + 1$
  $N_f = 2$
- expansion coefficients $c_i(T)$: $c_2 \quad \rightarrow \quad c_4, c_6$
  pronounced behaviour at $T_c \quad \Longleftarrow \quad \left.\frac{\partial^n G^2}{\partial \mu^n}\right|_{\mu=0}$
- from QCD to QPM:
  - ghost free gauge
  - 2-loop order in $\Phi \Longrightarrow s' = 0$
  - neglect D_L, S_- modes
  - gauge invariant HTL-expressions
  - neglect Landau damping & imaginary parts in Π, Σ
  - neglect dependence of $\hat{\Pi}_T$, $\hat{\Sigma}_+$ on energy and momentum
- under consideration: implications on viscosity ?